Navigation with Cellular CDMA Signals – Part I:
Signal Modeling and Software-Defined Receiver Design

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Abstract—A software-defined receiver (SDR) for navigation using cellular code-division multiple access (CDMA) signals is presented. The cellular forward-link signal structure is described and models for the transmitted and received signals are developed. Particular attention is paid to relevant information that could be extracted and subsequently exploited for positioning and timing purposes. The pseudorange from the proposed receiver is modeled and the pseudorange error is studied in an additive white Gaussian channel. Experimental results with aerial and ground vehicles utilizing the proposed SDR are presented demonstrating a close match between the variation in pseudoranges and the variation in true ranges between the receiver and two cellular CDMA base transceiver stations (BTSs). Moreover, the dynamics of the discrepancy between the observed clock biases of different sectors of the same BTS cell is modeled and validated experimentally. The consistency of the obtained model is analyzed through experimental tests in different locations, at different times, and for different cellular providers.

Index Terms—Radionavigation, signals of opportunity, opportunistic navigation, direct-sequence code-division multiple access, software radio, system identification.

I. INTRODUCTION

Traditional approaches to enable navigation in global navigation satellite system (GNSS)-challenged environments (e.g., indoors, deep urban canyons, and intentionally jammed and spoofed environments) have focused on coupling GNSS receivers with inertial navigation systems and advanced signal processing algorithms [1]–[4]. Recently, considerable attention has been devoted to exploiting ambient radio frequency (RF) signals of opportunity (SOPs) as a stand-alone alternative to GNSS or to complement GNSS-based navigation [5]–[8].

Different studies have been conducted for specific types of SOPs including AM/FM radio [9], [10], iridium satellites [11], [12], digital television (DTV) [13], [14], cellular [15]–[19], and Wi-Fi [20]–[22]. It has been demonstrated that AM signals could potentially provide 20 meter positioning accuracy [9]. A better localization performance could be achieved using DTV signals, where the average positioning error becomes less than 4 meters in certain favorable environments [13]. Experimental results for navigation using cellular code-division multiple access (CDMA) fused with DTV signals showed a navigation solution within 2 meters from that of a GPS solution and a maximum difference of 12 meters [17]. SOPs have also been used for indoor positioning, where it has been shown that an average positioning error of 4 meters could be achieved by coupling Wi-Fi and inertial measurement units (IMUs) in a SLAM framework [20]. Coupling observables from other signals such as GSM, digital audio broadcasting, and cellular 3G with IMU measurements also showed promising results [6]. Moreover, iridium satellite signals were considered to improve navigation performance in deep urban and indoor environments [23]. SOPs were also employed in timing applications, such as enabling longer integration time for GPS-assisted femtocells in indoor environments [24]. Besides these experimental studies, the literature on SOPs answers theoretical questions on the observability and estimability of the SOP signal landscape [25], [26], motion planning in the SOP landscape for optimal information gathering [27]–[29], and collaborative SOP landscape map building [30], [31].

There are three main challenges associated with using SOPs for navigation: (1) the unavailability of appropriate low-level signal models for optimal extraction of states and parameters of interest for positioning and timing purposes, (2) the absence of published receiver architectures capable of producing navigation observables, and (3) the lack of sources of error identification and error models for SOP-based navigation. To the authors’ knowledge, while previous work demonstrated experimental results for navigation via cellular CDMA signals, none of these three challenges has been fully addressed. This paper, the first in a series of two, addresses these three challenges for cellular CDMA signals. Cellular CDMA signals are particularly attractive SOPs due to their abundance, high carrier frequency, large bandwidth, high received power, and CDMA modulation structure, which is similar to the well-studied GPS signals.

Unlike GNSS signals, cellular CDMA signals are not intended for navigation [32]. As such, to exploit these signals for navigation purposes, the received signals must be parameterized in terms of relevant navigation observables. Subsequently, an appropriate specialized receiver capable of extracting this relevant positioning and timing information from the received signals must be designed. The navigation observables produced by these receivers can be used to either (1) map the states of the transmitting base transceiver station (BTS) tower (i.e., estimate the BTS’s position, clock bias, and...
Cellular CDMA communication receivers are routinely implemented in hardware in mobile phones; however, hardware implementations limit the ability to extract or modify information within the receiver. As such, a software-defined receiver (SDR) becomes an attractive platform of choice for implementing a cellular CDMA receiver for navigation purposes, because of its inherent advantages: (1) flexibility: designs are hardware independent, (2) modularity: different functions can be implemented independently, and (3) upgradability: minimal changes are needed to improve designs. Although most SDRs used to be limited to post-processing applications, processor-specific optimization techniques allow for real-time operation [33]. Consequently, SDR implementations are becoming more prevalent. Moreover, graphical programming languages such as LabVIEW and Simulink offer the advantage of a one-to-one correspondence between the architectural conceptualization of the SDR and software implementation [34]. An SDR for navigation with cellular CDMA signals based on the IS-95 standard was presented in [18].

Sources of error and the so-called error budget for GNSS-based navigation have been thoroughly studied [35], [36]. In contrast, navigation sources of error for SOPs are not yet fully characterized. It is important to note that while some of these errors are not harmful for communication purposes, they severely degrade the navigation performance if they are not modeled and accounted for appropriately. In [18], a new navigation error source corresponding to cellular CDMA signals was revealed, namely bias mismatch for different sectors within the same BTS cell. A rudimentary random walk (RW) model for the dynamics of this error was identified in [37]. This bias discrepancy across different sectors can be particularly harmful for navigation purposes in two scenarios. In the first scenario, a receiver that has no knowledge of its own states, nor has access to GNSS, is present in a cellular CDMA environment and is making pseudorange measurements to BTSs nearby. The receiver has access to estimates of the BTSs’ states through a central database. These estimates could be produced through a stationary mapping receiver or crowdsourced from mobile receivers in the environment. In some cases, while estimates of the BTS sector in which the navigating receiver is located may not be available, estimates of a different sector of the same BTS cell may be available in the database. If the navigating receiver uses such estimates without accounting for the fact that they are produced by a mapping receiver in a different sector, the discrepancy between the sector clock biases will introduce errors on the order of tens of meters in the receiver’s position estimate and tens of nanoseconds in the receiver’s clock bias estimate. A second scenario where this discrepancy must be accounted for is when the receiver is navigating in a simultaneous localization and mapping (SLAM) framework. In the SLAM approach, the receiver does not need access to the BTS state estimates from an external source; however, it must account for the aforementioned discrepancy when transitioning from one sector of the BTS to another sector.

This paper makes four contributions. First, it extends the work in [18] by presenting precise, low-level signal models for optimal extraction of relevant navigation and timing information from received cellular CDMA signals compatible with the latest cdma2000 standard. Second, the statistics of the pseudorange error in an additive white Gaussian channel are derived. Third, the paper presents experimental results validating this SDR by comparing the variation in the pseudoranges obtained by the proposed SDR and the true ranges to two BTSs. Fourth, the paper identifies an elaborate exponentially correlated dynamical model for the discrepancy in the clock biases in different sectors of a BTS cell and discusses when this model could be appropriately approximated by a RW model. The derived model is validated experimentally in different locations, at different times, and for different cellular providers.

The remainder of the paper is organized as follows. Section II provides an overview of the cellular CDMA forward link pilot signal structure. Section III presents a complete implementation of the acquisition and tracking stages of a navigation cellular CDMA SDR. Section IV analyzes the statistics of the pseudorange error of the CDMA SDR in an additive white Gaussian channel. Section V models the discrepancy between the clock biases of different sectors of the same BTS. Section VI validates the proposed navigation SDR and analyzes the consistency of the obtained clock bias discrepancy model experimentally. Concluding remarks are given in Section VII.

II. CELLULAR CDMA FORWARD LINK SIGNAL STRUCTURE

In a cellular CDMA communication system, several logical channels are multiplexed on the forward link channel, including: a pilot channel, a sync channel, and 7 paging channels [38]. The following subsection presents an overview of the modulation process of the forward link pilot channel and provides models of the transmitted and received signals from which timing and positioning information can be extracted.

A. Modulation of Forward Link CDMA Pilot Signals

The data transmitted on the forward link channel in cellular CDMA systems (i.e., BTS to mobile receiver) is modulated through quadrature phase shift keying (QPSK) and then spread using direct-sequence CDMA (DS-CDMA). The in-phase and quadrature components, I and Q, respectively, of the pilot channel carry the same message \( m(t) \). The spreading sequences \( c_I \) and \( c_Q \), called the short code, are 2\(^15\)-chip long pseudorandom noise (PN) codes that are generated using linear feedback shift registers (LFSRs). In order to distinguish the received data from different BTSs, each station uses a shifted version of the PN codes. This shift, known as the PN offset, is unique for each BTS and is an integer multiple of 64 chips, hence a total of 512 PN offsets can be realized. It can be shown that the cross-correlation of the same PN sequence with different pilot offsets is negligible [32], [39]. The transmitted pilot signal is nothing but the short code; however, other channels, such as the sync and paging channels, carry data and are furthermore spread by Walsh codes. The CDMA signal is subsequently filtered using a digital pulse shaping filter that limits the bandwidth of the transmitted CDMA signal.
according to the cdma2000 standard. The signal is finally modulated by the carrier frequency $\omega_c$ to produce $s(t)$.

B. Transmitted Signal Model

The transmitted pilot signal $s(t)$ by a particular BTS can be expressed as

$$s(t) = \sqrt{C} \{ c_I^t [t - \Delta(t)] \cos(\omega_c t) - c_Q^t [t - \Delta(t)] \sin(\omega_c t) \}$$

$$= \sqrt{C} \{ c_I^t [t - \Delta(t)] + j c_Q^t [t - \Delta(t)] \} \cdot e^{j \omega_c t}$$

$$+ \sqrt{C} \{ c_I^t [t - \Delta(t)] - j c_Q^t [t - \Delta(t)] \} \cdot e^{-j \omega_c t},$$

where $C$ is the total power of the transmitted signal; $c_I^t(t) = c_I(t) \ast h(t)$ and $c_Q^t(t) = c_Q(t) \ast h(t)$; $h(t)$ is the continuous-time impulse response of the pulse shaping filter; $c_I$ and $c_Q$ are the in-phase and quadrature PN sequences, respectively; $\omega_c = 2\pi f_c$ with $f_c$ being the carrier frequency; and $\Delta$ is the absolute clock bias of the BTS from GPS time. The total clock bias $\Delta$ is defined as

$$\Delta(t) = 64 \cdot (PN_{\text{offset}} T_c + \delta t_s(t),$$

where $PN_{\text{offset}}$ is the PN offset of the BTS, $T_c = \frac{10^{-6}}{12288}$ s is the chip interval, and $\delta t_s$ is the BTS clock bias. Since the chip interval is known and the PN offset can be decoded by the receiver, only $\delta t_s$ needs to be estimated. While the clock bias of the BTS can be neglected for communication purposes, it cannot be ignored for navigation purposes and must be estimated in some fashion. The sequel to this paper presents a framework for estimating this clock bias that is based on mapping and navigating receivers.

C. Received Signal Model After Front-End Processing

Assuming the transmitted signal to have propagated through an additive white Gaussian noise channel with a power spectral density of $\frac{W}{2}$, a model of the received discrete-time (DT) signal $r[m]$ after RF front-end processing: downmixing, a quadrature approach to bandpass sampling [40], and quantization can be expressed as

$$r[m] = \sqrt{C} \{ c_I^t [mT_s - t_s(mT_s)] - j c_Q^t [mT_s - t_s(mT_s)] \} e^{j \theta(t_m) + n(m]}$$

where $t_s(mT_s) \triangleq \delta t_{\text{TOF}} + \Delta(t_k - \delta t_{\text{TOF}})$ is the PN code phase of the BTS, $mT_s$ is the sample time in receiver time, $T_s$ is the sampling period, $\delta t_{\text{TOF}}$ is the time-of-flight (TOF) from the BTS to the receiver, $\theta$ is the beat carrier phase of the received signal, and $n(m) = n_I[m] + j n_Q[m]$ with $n_I$ and $n_Q$ being independent, identically-distributed (i.i.d.) Gaussian random sequences with zero-mean and variance $\frac{\sigma_n}{2}$. The receiver developed in Section III will operate on the samples of $r[m]$ in (1).

III. CELLULAR CDMA NAVIGATION RECEIVER ARCHITECTURE

The cellular CDMA navigation receiver consists of three main stages: signal acquisition, tracking, and decoding. The proposed receiver will utilize the pilot signal to detect the presence of a CDMA signal and then track it, as will be discussed in this section. The next subsection gives a brief description of the correlation process in the cellular CDMA navigation receiver. The following subsections present a software implementation in LabVIEW of the cellular CDMA navigation receiver. The next subsection gives a brief description of the correlation process in the cellular CDMA navigation receiver. The following subsections present a software implementation in LabVIEW of the cellular CDMA navigation receiver.
short \(T_{\text{sub}}\). Substituting for \(r[m]\) and \(x[m]\), defined in (1)-(2), into (3), it can be shown that
\[
Z_k = \sqrt{C} R_c(\Delta t_k) \left[ \frac{1}{N_s} \sum_{m=k}^{k+N_s-1} e^{j \Delta \theta(t_m)} \right] + n_k, \quad (4)
\]
where \(R_c\) is the autocorrelation function of the PN sequences \(c' \) and \(c'_Q\), \(\Delta t_k \approx \hat{t}_{sk} - t_{sk}\) is the code phase error, \(\Delta \theta(t_m) \approx \theta(t_m) - \theta(t_n)\) is the carrier phase error, and \(n_k \approx n_{tk} + j n_{Qk}\) with \(n_{tk}\) and \(n_{Qk}\) being i.i.d. Gaussian random sequences with zero-mean and variance \(\frac{N}{2T_{\text{sub}}} n_s = \frac{N}{2T_s}\).

**B. Acquisition**

The objective of this stage is to determine which BTSs are in the receiver’s proximity and to obtain a coarse estimate of their corresponding code start times and Doppler frequencies. A search over the code start time and Doppler frequency is performed to detect the presence of a signal. Based on experimental data, the Doppler frequency search window is chosen to be between -500 and 500 Hz at a carrier frequency in the 800/850 MHz cellular band, with a frequency spacing \(\Delta f_D\) between 8 and 12 Hz if \(T_{\text{sub}}\) is only one PN code period. The code start time search window is naturally chosen to be one PN code interval with a delay spacing of one sample. The proposed receiver performs a parallel code phase search by exploiting the optimized efficiency of the fast Fourier transform (FFT) [43]. A hypothesis test on \(|Z_k|^2\) could be performed to decide whether the peak corresponds to a transmitted signal or to noise. Since there is only one PN sequence, the search needs to be performed once. Fig. 3(a) illustrates the front panel of the acquisition stage of the LabVIEW cellular CDMA SDR showing \(|Z_k|^2\) along with \(\hat{t}_{sk}, f_{Dk}, \) PN offset, and carrier-to-noise ratio \(C/N_0\) for a particular BTS.

**C. Tracking**

After obtaining an initial coarse estimate of the code start time and Doppler frequency, the receiver refines and maintains these estimates via tracking loops. In the proposed design, a phase-locked loop (PLL) is employed to track the carrier phase and a carrier-aided delay-locked loop (DLL) is used to track the code phase. The PLL and DLL are discussed next.

1) PLL: The PLL consists of a phase discriminator, a loop filter, and a numerically-controlled oscillator (NCO). Since the receiver is tracking the data-less pilot channel, an atan2 discriminator, which remains linear over the full input error range of \(\pm \pi\), could be used without the risk of introducing phase ambiguities. It was found that the receiver could easily track the carrier phase with a second-order PLL with a loop filter transfer function given by
\[
F_{\text{PLL}}(s) = \frac{2K\omega_n s + \omega_n^2}{s}, \quad (5)
\]
where \(K \equiv \frac{1}{\sqrt{2}}\) is the damping ratio and \(\omega_n\) is the undamped natural frequency, which can be related to the PLL’s noise-equivalent bandwidth \(B_{n,\text{PLL}}\) by \(B_{n,\text{PLL}} = \frac{\omega_n}{8K}(4\zeta^2 + 1)\) [36]. The output of the loop filter \(v_{\text{PLL}}\) is the rate of change of the carrier phase error, expressed in rad/s. The Doppler frequency is deduced by dividing \(v_{\text{PLL}}\) by \(2\pi\). The loop filter transfer function in (5) is discretized at a sampling period \(T_{\text{sub}}\) and realized in state-space. The noise-equivalent bandwidth is chosen to range between 4 and 8 Hz.

2) DLL: The carrier-aided DLL employs the non-coherent dot product discriminator. In order to compute the code phase error, the dot product discriminator uses the prompt, early, and late correlations, denoted by \(Z_{pk}, Z_{rk},\) and \(Z_{lk}\), respectively. The prompt correlation was described in Subsection III-A. The early and late correlations are calculated by correlating the received signal with an early and a delayed version of the prompt PN sequence, respectively. The time shift between \(Z_{ek}\) and \(Z_{lk}\) is defined by an early-minus-late time \(t_{\text{eml}}\), expressed in chips. Since the autocorrelation function of the transmitted cellular CDMA pulses is not triangular as in the case of GPS, a wider \(t_{\text{eml}}\) is preferable in order to have a significant difference between \(Z_{pk}, Z_{rk},\) and \(Z_{lk}\). Fig. 1 shows the autocorrelation function of the cellular CDMA PN code as specified by the cdma2000 standard and that of the C/A code in GPS. It can be seen from Fig. 1 that for \(t_{\text{eml}} \leq 0.5\) chips, \(R_c(\tau)\) in the cdma2000 standard has approximately a constant value, which is not desirable for precise tracking. In this paper, a \(t_{\text{eml}}\) of 1 to 1.2 chips is chosen.

![Fig. 1. Autocorrelation function of GPS C/A code and cellular CDMA PN sequence according to the cdma2000 standard.](image)

The DLL loop filter is a simple gain \(K\), with a noise-equivalent bandwidth \(B_{n,\text{DLL}} = \frac{K}{T_s} \approx 0.5\) Hz. The output of the DLL loop filter \(v_{\text{DLL}}\) is the rate of change of the code phase, expressed in s/s. Assuming low-side mixing, the code start time is updated according to
\[
\hat{t}_{sk+1} = \hat{t}_{sk} - (v_{\text{DLL},k} + \hat{f}_{Dk}/f_c) \cdot N_sT_s.
\]

The pseudorange estimate \(\rho\) can therefore be deduced by multiplying the code start time by the speed-of-light \(c\), i.e.,
\[
\rho(k) = c \cdot \hat{t}_{sk}. \quad (6)
\]

Fig. 2 depicts a diagram of the tracking loops. Fig. 3(b)–(e) shows the intermediate signals produced within the tracking loops of the LabVIEW cellular CDMA navigation receiver: phase error, code error, Doppler frequency, and pseudorange.

In the next section, the tracking performance of the DLL is studied and the closed-loop statistics of the code start time estimate are derived.
time-update error equation [44]
\[ \Delta t_{k+1} = (1 - 4B_{n,DLL}T_{\text{sub}})\Delta t_k + KD_k, \]
where \( D_k \) is the output of the code discriminator. The discriminator statistics are discussed next.

A. Discriminator Statistics

In order to study the discriminator statistics, the received signal noise statistics must first be determined. In what follows, the received signal noise is characterized for an additive white Gaussian channel.

1) Received Signal Noise Statistics: In order to make the analysis more tractable, the continuous-time (CT) received signal and correlation are considered. The transmitted signal is assumed to propagate in an additive white Gaussian noise channel with a power spectral density \( \frac{N_0}{2} \). The CT received signal after downmixing and bandpass sampling is given by
\[ r(t) = \frac{\sqrt{C}}{2} [c'_I(t - t_s) - jc'_Q(t - t_s)] e^{j\theta(t)} + n(t), \]
and the CT matched-filtered baseband signal \( x(t) \) is given by
\[ x(t) = [r(t) \cdot e^{-j\theta(t)}] + h(-t). \]

The resulting early and late correlations in the DLL are given by
\[ Z_{e_k} = \int_0^{T_{\text{sub}}} x(t) [c_I(t - \tau_{e_k}) + jc_Q(t - \tau_{e_k})] \, dt, \]
\[ Z_{l_k} = \int_0^{T_{\text{sub}}} x(t) [c_I(t - \tau_{l_k}) + jc_Q(t - \tau_{l_k})] \, dt, \]
where \( \tau_{e_k} \triangleq \hat{t}_{e_k} - \frac{\tau_{e_m}}{2}T_c \) and \( \tau_{l_k} \triangleq \hat{t}_{l_k} + \frac{\tau_{l_m}}{2}T_c \). Assuming the receiver is closely tracking the carrier phase [36], the early and late correlations may be approximated with
\[ Z_{e_k} \approx T_{\text{sub}}\sqrt{C}R_c(\Delta t_k - \frac{\tau_{e_m}}{2}T_c) + n_{e_k} \triangleq S_{e_k} + n_{e_k}, \]
\[ Z_{l_k} \approx T_{\text{sub}}\sqrt{C}R_c(\Delta t_k + \frac{\tau_{l_m}}{2}T_c) + n_{l_k} \triangleq S_{l_k} + n_{l_k}, \]
where \( n_{e_k} \) and \( n_{l_k} \) are zero-mean Gaussian random variables with the following variances and covariances
\[ \text{var}\{n_{e_k}\} = \text{var}\{n_{l_k}\} = \frac{T_{\text{sub}}N_0}{2}, \quad \forall k, \]
\[ \mathbb{E}\{n_{e_k}n_{l_k}\} = \frac{T_{\text{sub}}N_0R_c(T_{\text{eml}}T_c)}{2}, \quad \forall k, \]
\[ \mathbb{E}\{n_{e_k}n_{e_j}\} = \mathbb{E}\{n_{l_k}n_{l_j}\} = \mathbb{E}\{n_{e_k}n_{l_j}\} = 0, \quad \forall k \neq j. \]

2) Coherent Discriminator Statistics: The coherent baseband discriminator function is defined as
\[ D_k \triangleq \frac{Z_{e_k} - Z_{l_k}}{\sqrt{C}} = \frac{(S_{e_k} - S_{l_k})}{\sqrt{C}} + \frac{(n_{e_k} - n_{l_k})}{\sqrt{C}}. \]
The normalized signal component of the discriminator function \( \frac{(S_{e_k} - S_{l_k})}{T_{\text{sub}}\sqrt{C}} \) is shown in Fig. 4 for \( T_{\text{eml}} = \{0.25, 0.5, 1, 1.5, 2\} \).
It can be seen from Fig. 4 that for small values of $\frac{\Delta t_k}{T_c}$, the discriminator function can be approximated by a linear function given by

$$D_k \approx \alpha \Delta t_k + \frac{(n_{ck} - n_{kl})}{\sqrt{C}},$$

where $\alpha$ is the slope of the discriminator function at $\Delta t_k = 0$ [44], which is obtained by

$$\alpha = \left. \frac{\partial D_k}{\partial \Delta t_k} \right|_{\Delta t_k=0} = T_{sub} \left[ \frac{d}{d\tau} R_c(\tau) - \frac{d}{d\tau} R_c(\tau) \right] \bigg|_{\tau = \frac{t_{eml}}{T_c}}.$$

Since $R_c(\tau)$ is symmetric,

$$\left. \frac{d}{d\tau} R_c(\tau) \right|_{\tau = \frac{t_{eml}}{T_c}} = \left. \frac{d}{d\tau} R_c(\tau) \right|_{\tau = -\frac{t_{eml}}{T_c}} \triangleq R_c(\frac{t_{eml}}{2} T_c),$$

and the linearized discriminator output becomes

$$D_k \approx 2T_{sub} R_c' \left( \frac{t_{eml}}{2} T_c \right) \Delta t_k + \frac{(n_{ck} - n_{kl})}{\sqrt{C}}. \quad (8)$$

It is worth noting that $R_c(\tau)$ and $R_c'(\tau)$ are obtained by numerically computing the autocorrelation function of the pulse-shaped short code. Since the FIR of the pulse-shaping filter $h[k]$ is defined over only 48 values of $k$, the autocorrelation function $R_c(\tau)$ will be defined over 95 values of $\tau$. However, interpolation may be used to evaluate $R_c(\tau)$ and $R_c'(\tau)$ at any $\tau$. The mean and variance of $D_k$ can be obtained from (8), and are given by

$$\mathbb{E}\{D_k\} = 2T_{sub} R_c' \left( \frac{t_{eml}}{2} T_c \right) \Delta t_k, \quad (9)$$

$$\text{var}\{D_k\} = \frac{1}{C} \text{var}\{n_{ck} - n_{kl}\}$$

$$= \frac{1}{C} \left[ \text{var}\{n_{ck} + n_{kl}\} - 2\mathbb{E}\{n_{ck} n_{kl}\} \right]$$

$$= T_{sub} N_0 \left[ 1 - R_c(\frac{t_{eml}}{T_c}) \right]. \quad (10)$$

Now that the discriminator statistics are known, the closed-loop pseudorange error is characterized.

B. Closed-Loop Analysis

In order to achieve the desired loop noise-equivalent bandwidth, $K$ in (7) must be normalized according to

$$K = \frac{4B_{n, DLL} T_{sub} \Delta t_k}{\mathbb{E}\{D_k\} \bigg|_{\Delta t_k=0} = 2B_{n, DLL} \frac{R_c(\frac{t_{eml}}{2} T_c)}{\Delta t_k}}. \quad (11)$$

In cellular CDMA systems, for a $t_{eml}$ of 1.2, the loop filter gain becomes $K \approx 4B_{n, DLL}$, hence the choice of $K$ in Subsection III-C. Assuming a zero-mean tracking error, i.e., $\mathbb{E}\{\Delta t_k\} = 0$, then the variance of the code start time error is given by

$$\text{var}\{\Delta t_{k+1}\} = (1 - 4B_{n, DLL} T_{sub})^2 \text{var}\{\Delta t_k\} + K^2 \text{var}\{D_k\}. \quad (12)$$

At steady-state, $\text{var}\{\Delta t_{k+1}\}$ becomes

$$\text{var}\{\Delta t_{k+1}\} = \text{var}\{\Delta t_k\} = \text{var}\{\Delta \tau\}, \quad (13)$$

where $\Delta \tau$ is the steady-state code start time error. Combining (11)–(13) yields

$$\text{var}\{\Delta \tau\} = \frac{B_{n, DLL} \rho(t_{eml})}{2(1 - 2B_{n, DLL} T_{sub}) C/N_0}, \quad (14)$$

where

$$\rho(t_{eml}) \triangleq 1 - R_c(\frac{t_{eml} T_c}{2}).$$

The pseudorange can hence be expressed as

$$\rho(k) = c \cdot t_{sk} + c \cdot \Delta t_k \triangleq c \cdot t_{sk} + v(k),$$

where $v(k)$ is a zero-mean random variable with variance $\sigma^2 = c^2 \cdot \text{var}\{\Delta \tau\}$. Fig. 5 shows a plot of $\sigma$ as a function of the carrier-to-noise ratio $\frac{C}{N_0}$ for $t_{eml} = 1.25$ chips.

V. CLOCK BIAS DISCREPANCY MODEL BETWEEN DIFFERENT SECTORS OF A BTS CELL

A typical CDMA BTS transmits into three different sectors within a particular cell. Ideally, all sectors’ clocks should be driven by the same oscillator, which implies that the same clock bias (after correcting for the PN offset) should be observed in all sectors of the same cell. However, factors such as unknown distance between the phase-center of the sector antennas, delays due to RF connectors and other components (e.g., cabling, filters, amplifiers, etc.) cause the clock biases corresponding to different BTS sectors to be slightly different. This behavior was consistently observed experimentally in different locations, at different times, and for different cellular providers [18], [45]. In this section, the model for the pseudorange produced by the cellular CDMA navigation receiver developed in Section III is given. Subsequently, a stochastic dynamic model for the observed clock bias mismatch for different sectors of the same BTS cell is identified and experimentally validated.
A. Pseudorange Measurement Model

The pseudorange can be obtained from the proposed cellular CDMA navigation SDR by multiplying the code phase estimate by the speed-of-light. A model for this produced pseudorange can be parameterized as a function of the receiver and BTS position and clock bias states. For simplicity, a planar environment will be assumed, with the receiver and BTS three-dimensional (3-D) position states appropriately projected onto such planar environment. The subsequent discussion can be straightforwardly generalized to 3-D. The state of the receiver is defined as \( x_r = [r_r, c \delta t_r]_T \), where \( r_r = [x_r, y_r]^T \) is the position vector of the navigator, \( \delta t_r \) is the navigator’s clock bias, and \( c \) is the speed-of-light. Similarly, the state of the \( i \)th BTS is defined as \( x_{si} = [r_{si}, c \delta t_{si}]^T \), where \( r_{si} = [x_{si}, y_{si}]^T \) is the position vector of the \( i \)th BTS and \( \delta t_{si} \) is the clock bias. After mild approximations discussed in [26], the pseudorange measurement to the \( i \)th BTS at time \( k \), \( \rho_i(k) \), can be expressed as

\[
\rho_i(k) = \|r_{si}(k) - r_{si}\| + c \cdot [\delta t_{ri}(k) - \delta t_{si}(k)] + v_i(k),
\]

where \( v_i \) is the observation noise, which is modeled as a zero-mean white Gaussian random sequence with variance \( \sigma_i^2 \).

B. Sector Clock Bias Discrepancy Detection

In order to detect the discrepancy between sectors’ clock biases, the proposed cellular CDMA receiver was placed at the border of two sectors of a BTS cell and was drawing pseudorange measurements from both sector antennas. The receiver had full knowledge of its state and of the BTS’s position. Subsequently, the receiver solved for the BTS clock biases \( \delta t_{s1}^{(p)} \) and \( \delta t_{s2}^{(q)} \) observed in sectors \( p_i \) and \( q_i \), respectively. A realization of \( \delta t_{s1}^{(p)} \) and \( \delta t_{s2}^{(q)} \) is depicted in Fig. 6.

![Fig. 6. (a) A cellular CDMA receiver placed at the border of two sectors of a BTS cell, making pseudorange observations on both sector antennas simultaneously. The receiver has knowledge of its own states and has knowledge of the BTS position states. (b) Observed BTS clock bias corresponding to two different sectors from a real BTS (Verizon Wireless).](image)

Fig. 6 suggests that the clock biases \( \delta t_{s1}^{(p)} \) and \( \delta t_{s2}^{(q)} \) can be related through

\[
\delta t_{s1}^{(q)}(k) = \delta t_{s1}^{(p)}(k) + [1 - 1_{q_i}(p_i)] \epsilon_i(k),
\]

where \( \epsilon_i \) is a random sequence that models the discrepancy between the sectors’ clock biases and

\[
1_{q_i}(p_i) = \begin{cases} 1, & \text{if } p_i = q_i, \\ 0, & \text{otherwise}, \end{cases}
\]

is the indicator function.

**Remark** The cdma2000 protocol requires all PN offsets to be synchronized to within \( 10 \mu s \) from GPS time; however, synchronization to within \( 3 \mu s \) is recommended [46]. Since each sector of a BTS uses a different PN offset, then the clock biases \( \delta t_{s1}^{(p)} \) and \( \delta t_{s2}^{(q)} \) will be bounded according to \( -10 \mu s \leq \delta t_{s1}^{(p)}(k) \leq 10 \mu s \) and \( -10 \mu s \leq \delta t_{s2}^{(q)}(k) \leq 10 \mu s \). Therefore, \( \epsilon_i \) will be within \( 20 \mu s \) from GPS time, namely

\[-20 \mu s \leq \epsilon_i \leq 20 \mu s.\]

The discrepancy \( \epsilon_i^2 \) between the clock biases observed in two different sectors of some BTS cell over a 24-hour period is shown in Fig. 7(a)-(b) for two different BTSs. Both cellular towers pertain to the U.S. cellular provider Verizon Wireless and are located near the University of California, Riverside campus. The cellular signals were recorded between September 23 and 24, 2016. It can be seen from Fig. 7 that \( |\epsilon_i| \) is bounded by approximately 2.02\( \mu s \) and 0.65\( \mu s \), respectively, which is well below 20\( \mu s \).

![Fig. 7. The discrepancies \( \epsilon_1 \) and \( \epsilon_2 \) between the clock biases observed in two different sectors of some BTS cell over a 24-hour period. (a) and (b) correspond to \( \epsilon_1 \) and \( \epsilon_2 \) for BTSs 1 and 2, respectively. Both BTSs pertain to the U.S. cellular provider Verizon Wireless and are located near the University of California, Riverside campus. The cellular signals were recorded between September 23 and 24, 2016. It can be seen that \( |\epsilon_i| \) is well below 20\( \mu s \).](image)

In what follows, a stochastic dynamic model for \( \epsilon_i \) is identified.

C. Model Identification

It is hypothesized that the discrepancy \( \epsilon_i(k) = \delta t_{s1}^{(q)}(k) - \delta t_{s1}^{(p)}(k) \) for \( p_i \neq q_i \) adheres to an autoregressive (AR) model of order \( n \) [47], which can be expressed as

\[
\epsilon_i(k) + \sum_{j=1}^{n} a_j \epsilon_i(k-j) = \zeta_i(k),
\]

where \( \zeta_i \) is a white sequence. The objective is to find the order \( n \) and the coefficients \( \{a_j\}_{j=1}^{n} \) that will minimize the sum of the squared residuals \( \sum_{k=0}^{\infty} \zeta_i^2(k) \). To find the order \( n \), several AR models were identified and for a fixed order,
a least-squares estimator was used to solve for \( \{a_{ij}\}_{j=1}^n \). It was noted that the sum of the squared residuals corresponding to each \( n \in \{1, \ldots, 10\} \) were comparable, suggesting that the minimal realization of the AR model is of first-order. For \( n = 1 \), it was found that \( a_{11} = -(1 - \beta_i) \), where \( 0 < \beta_i < 1 \) (on the order of \( 8 \times 10^{-5} \) to \( 3 \times 10^{-4} \)). This implies that \( \epsilon_i \) is an exponentially correlated random variable (ECRV) with the continuous-time (CT) dynamics given by

\[
\dot{\epsilon}_i(t) = -\alpha_i \epsilon_i(t) + \xi_i(t),
\]

where \( \alpha_i \triangleq \frac{1}{\tau_i} \), \( \tau_i \) is the time constant of the discrepancy dynamical model, and \( \xi_i \) is a CT white process with variance \( \sigma^2_{\xi_i} \). Discretizing (16) at a sampling period \( T \) yields the DT model

\[
\epsilon_i(k + 1) = \phi_k \epsilon_i(k) + \zeta_i(k),
\]

where \( \phi_k = e^{-\alpha_i T} \). The variance of \( \zeta_i \) is given by \( \sigma^2_{\zeta_i} = \sigma^2_{\xi_i} \left(1 - e^{-2\alpha_i T}\right) \). Fig. 8 shows an experimental realization of \( \epsilon_i \) and the corresponding residual \( \zeta_i \).

![Fig. 8. (a) A realization of the discrepancy \( \epsilon_i \) between the observed clock biases of two BTS sectors and (b) the corresponding residual \( \zeta_i \).](image)

D. Model Validation

The identified model in (17) was validated through residual analysis [47]. To this end, the autocorrelation function (acf) and power spectral density (psd) of the residual error \( \epsilon_i \) defined as the difference between the measured data \( \epsilon'_i \) and predicted value from the identified model \( \epsilon_i \) in (17), i.e., \( \epsilon_i \triangleq \epsilon'_i - \epsilon_i \), were computed. Fig. 9 shows the acf and psd of \( \epsilon_i \) computed from a different realization of \( \epsilon_i \). The psd was computed using Welch’s method [48]. It can be seen from Fig. 9 that the residual error \( \epsilon_i \) is nearly white; hence, the identified model is capable of describing the true system.

![Fig. 9. The (a) acf and (b) psd of \( \epsilon_i \) with a sampling frequency of 5 Hz.](image)

E. Residual Statistics Characterization

Next, the probability density function (pdf) of \( \zeta_i \) will be characterized, assuming that \( \zeta_i \) is an ergodic process. It was found that the Laplace distribution best matches the actual distribution of \( \zeta_i \) obtained from experimental data, i.e., the pdf of \( \zeta_i \) is given by

\[
p(\zeta_i) = \frac{1}{2\lambda_i} \exp \left(-\frac{|\zeta_i - \mu_i|}{\lambda_i}\right),
\]

where \( \mu_i \) is the mean of \( \zeta_i \) and \( \lambda_i \) is the parameter of the Laplace distribution, which can be related to the variance by \( \sigma^2_{\zeta_i} = 2\lambda^2_i \). A maximum likelihood estimator (MLE) was adopted to calculate the parameters \( \mu_i \) and \( \lambda_i \) of \( p(\zeta_i) \) [49]. Fig. 10 shows the actual distribution of the data along with the estimated pdf. For comparison purposes, a Gaussian and Logistic pdf fits obtained via an MLE are plotted as well.

![Fig. 10. Distribution of \( \zeta_i \) from experimental data and the estimated Laplace pdf via MLE. For comparison purposes, a Gaussian (dashed) and Logistic (dotted) pdf fits are plotted as well.](image)

It was noted that \( \mu_i \approx 0 \) from several batches of collected experimental data; therefore, \( \zeta_i \) is appropriately modeled as a zero-mean white Laplace-distributed random sequence with variance \( 2\lambda^2_i \).

F. Statistics of the Discrepancy Between Sector Clock Biases

The solution to the dynamic model (17) can be expressed as

\[
\epsilon_i(k) = \phi^k \epsilon_i(0) + \sum_{l=0}^{k-1} \phi^k \epsilon_i(l),
\]

where \( \epsilon_i(0) \) is the known initial discrepancy. Without loss of generality, \( \epsilon_i(0) \) is assumed to be zero. Therefore, \( \epsilon_i(k) \) has mean \( \mathbb{E}[\epsilon_i(k)] = 0 \) and variance \( \text{var}[\epsilon_i(k)] = \sigma^2_{\epsilon_i} \left(1 - e^{-2\alpha_i kT}\right) \). Note that the discrepancy \( \epsilon_i \) is the weighted sum of uncorrelated Laplace-distributed random variables. The central limit theorem asserts that the pdf of \( \epsilon_i \) converges to a Gaussian pdf. It was noted that the convergence happens for \( k \geq 9 \) for \( \phi_i \geq 0.95 \), as depicted in Fig. 11.

G. Approximation with a Random Walk

When \( \alpha_i \to 0 \), the dynamics of \( \epsilon_i(k) \) converge to that of a random walk. Since the values of \( \alpha_i \) obtained experimentally are very small, studying the RW model as an approximation becomes relevant. The mean of the RW process is also zero and the variance is given by \( \sigma^2_{\epsilon_i} kT \). It can be readily shown that

\[
\sigma^2_{\epsilon_i} kT > \sigma^2_{\epsilon_i} \left(1 - e^{-2\alpha_i kT}\right), \forall \alpha_i > 0, k > 0, \text{ and } T > 0.
\]

Denote the relative error between the variances of the ECRV and RW models by \( \gamma \), then the following can be established

\[
\frac{1}{2\alpha_i kT} \left(1 - e^{-2\alpha_i kT}\right) \geq 1 - \gamma.
\]
\[ f(x, \gamma) \geq 0, \]

where

\[ x = 2\alpha \gamma kT \quad \text{and} \quad f(x, \gamma) = 1 - (1 - \gamma)x - e^{-x}. \]

Fig. 12(a) shows \( f(x, \gamma) \) as a function of \( x \) for different values of \( \gamma \). Let \( x^* = g(\gamma) \) denote the solution to \( f(x, \gamma) = 0 \) for a given \( \gamma \). According to Fig. 12(a), for a given \( \gamma \), \( f(x, \gamma) \geq 0 \) is satisfied \( \forall x \in (0, g(\gamma)) \). Fig. 12(b) depicts the solution \( x^* = g(\gamma) \) as a function of \( \gamma \). Note that \( g(\gamma) \) does not have a closed form but can be calculated using iterative methods, e.g., Newton’s method.

Subsequently, for a desired \( \gamma \) and a known \( \alpha \), one can solve for \( k \) that guarantees the relative error between the RW and ECRV variances to be less than \( \gamma \) using \( 2\alpha \gamma kT \leq g(\gamma) \). For example, given that \( \gamma = 0.01 \) and \( \alpha = 3 \times 10^{-4} \text{Hz} \), then for \( kT \leq \frac{g(0.01)}{2 \alpha \gamma} = 33.55 \text{s} \), the relative error between the RW and ECRV variances will remain less than 1%.

VI. EXPERIMENTAL RESULTS

In this section, experimental results on an aerial and ground vehicle, validating the proposed cellular CDMA navigation SDR are presented. Next, the consistency of the clock bias discrepancy model derived in Section V is analyzed experimentally.

A. Cellular CDMA Navigation SDR Experimental Results

In order to test the proposed cellular CDMA SDR, the variation in the pseudorange obtained by the receiver was compared to the variation in true range between the moving receiver and cellular CDMA BTSs. For this purpose, two experiments are conducted where the proposed receiver was mounted on (1) an unmanned aerial vehicle (UAV) and (2) a ground vehicle.

1) UAV Results: In the first experiment, a DJI Matrice 600 UAV was equipped with the proposed SDR, a consumer-grade 800/1900 MHz cellular antenna, and a small consumer-grade GPS antenna to discipline the on-board oscillator. The cellular signals were down-mixed and sampled via a single-channel universal software radio peripheral (USRP) driven by a GPS-disciplined oscillator (GPSDO). The cellular receiver was tuned to a carrier frequency of 883.98 MHz, which is a channel allocated for the U.S. cellular provider Verizon Wireless. Samples of the received signals were stored for off-line post-processing. The cellular CDMA signals were processed by the proposed LabVIEW-based SDR. The ground-truth reference for the UAV trajectory was taken from its on-board navigation system, which uses GPS, an inertial navigation system, and other sensors. Fig. 13 shows the SOP BTS environment in which the UAV was present as well as the experimental hardware setup.

![SOP BTS environment and experimental hardware setup for the UAV experiment](image)

Fig. 13. SOP BTS environment and experimental hardware setup for the UAV experiment. Map data: Google Earth.

Over the course of the experiment, the receiver was listening to two BTSs, whose position states were mapped prior to the experiment according to the framework discussed in [31]. The distance \( D \) between the UAV and the BTS was calculated using the navigation solution produced by the UAV’s navigation system and the known BTS position, and the pseudorange \( \rho \) was obtained from the proposed cellular CDMA SDR mounted on the UAV over the trajectory shown in Fig. 14.

In order to validate the resulting pseudranges, the variation of the pseudorange \( \Delta \rho = \rho - \rho(0) \), where \( \rho(0) \) is the initial value of the pseudorange, and the variation in distance \( \Delta D = D - D(0) \), where \( D(0) \) is the initial distance between the UAV and the BTS are plotted in Fig. 15 for the two BTSs.

It can be seen from Fig. 15 that the variations in the pseudoranges follow closely the variations in distances. The difference between \( \Delta D \) and \( \Delta \rho \) for a particular BTS is due to the variation in the clock bias difference \( c(\delta t_r - \delta t_n) \) and the noise terms \( v_i \).
2) **Ground Vehicle Results:** In the second experiment, a car was equipped with the proposed SDR, a consumer-grade 800/1900 MHz cellular antenna, and a surveyor-grade GPS antenna to collect GPS L1 signal and to discipline the on-board oscillator. The cellular and GPS signals were down-mixed and synchronously sampled via a dual-channel USRP driven by a GPSDO. The cellular receiver was tuned to a carrier frequency of 882.75 MHz, which is also a channel allocated for the U.S. cellular provider Verizon Wireless. Samples of the received signals were stored for off-line post-processing. The cellular CDMA signals were processed by the proposed LabVIEW-based SDR. The GPS signal was processed by the Generalized Radionavigation Interfusion Device (GRID) SDR [50] and the resulting GPS solution was assumed to be the ground-truth reference for the car trajectory. Fig. 16 shows the SOP BTS environment, car trajectory, and the experimental hardware setup.

Over the course of the experiment, the receiver was listening to two BTSs, whose position states were mapped prior to the experiment according to the framework discussed in [31]. The change in the true range and the change in pseudorange are plotted in Fig. 17, similarly to the UAV experiment.

It can be seen from Fig. 17 that the variations in the pseudoranges follow closely the variations in distances. The difference between $\Delta D$ and $\Delta \rho$ for a particular BTS is due to the variation in the clock bias difference $c(\delta t_r - \delta t_s_i)$ and the noise terms $v_i$. The sequel paper will study the navigation performance and estimation of the clock bias in further detail.

### B. Clock Bias Discrepancy Model Consistency Analysis

The consistency of the clock bias discrepancy model was analyzed experimentally in different locations, at different times, and for different cellular providers. The results are presented in this section.

1) **Cellular CDMA SOP Test Scenarios and Hardware Setup:** The tests were performed twice at three different locations. There is a six-day period between each test at each of the three locations. A total of three carrier frequencies were considered, two of them pertaining to Verizon Wireless and one to Sprint. The test scenarios are summarized in Table II and Fig. 18. The date field in Table II shows the date in which the test was conducted in MM/DD/YYYY format.

<table>
<thead>
<tr>
<th>Test</th>
<th>Date</th>
<th>Location</th>
<th>Frequency</th>
<th>Provider</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>01/14/2016</td>
<td>1</td>
<td>882.75 MHz</td>
<td>Verizon</td>
</tr>
<tr>
<td>(b)</td>
<td>01/20/2016</td>
<td>1</td>
<td>882.75 MHz</td>
<td>Verizon</td>
</tr>
<tr>
<td>(c)</td>
<td>08/28/2016</td>
<td>2</td>
<td>883.98 MHz</td>
<td>Verizon</td>
</tr>
<tr>
<td>(d)</td>
<td>09/02/2016</td>
<td>2</td>
<td>883.98 MHz</td>
<td>Verizon</td>
</tr>
<tr>
<td>(e)</td>
<td>08/28/2016</td>
<td>3</td>
<td>1940.0 MHz</td>
<td>Sprint</td>
</tr>
<tr>
<td>(f)</td>
<td>09/02/2016</td>
<td>3</td>
<td>1940.0 MHz</td>
<td>Sprint</td>
</tr>
</tbody>
</table>

For the purpose of collecting data, a receiver that was placed close to the border of two sectors for each BTS was equipped with two antennas to acquire and track: (1) GPS signals and (2) signals from the cellular CDMA BTS sector antennas. The CDMA antenna used for the experiments in location 1...
was a consumer-grade 800/1900 MHz cellular antenna and a high-gain tri-band cellular antenna for locations 2 and 3. Both GPS antennas were surveyor-grade Leica antennas. The GPS and cellular signals were simultaneously down-mixed and synchronously sampled at 2.5 MS/s via a dual channel USRP driven by a GPSDO. Samples of the received signals were stored for off-line post-processing. The GPS signal was processed by GRID and the cellular CDMA signals were processed by the proposed LabVIEW-based SDR. The receiver’s clock bias obtained from the GPS solution was used to solve for the BTS sector clock bias. Fig. 19 shows the experimental hardware setup.

2) Analysis of Sector Clock Bias Discrepancy Realizations: Fig. 20 shows six realizations, five minutes each, of the discrepancy corresponding to Tests (a)–(f) in Table II. It can be seen from Fig. 20 that the behavior of the discrepancy is consistent across the tests. The initial discrepancy is subtracted out so that all realizations start at the origin. The inverse of the time constant for each realization was found to be \( \{\alpha_i\}_{i=1}^{6} = \{2.08, 1.66, 1.77, 1.70, 1.39, 2.53\} \times 10^{-4} \text{ Hz} \).

Next, the process noise driving the discrepancy is characterized. The process noise was calculated according to

\[
\zeta_i(k) = e_i(k + 1) - \phi_i e_i(k),
\]

where \( \phi_i = e^{-\alpha_i T} \) and \( T = 0.2s \). The acf of each of the six realizations of \( \zeta_i \) corresponding to the six realizations of \( e_i \) from Fig. 20 are shown in Fig. 21. Similarly to Fig. 9(a), the shape of the acfs in Fig. 21 exhibits very quick de-correlation, validating that \( \zeta_i \) is approximately a white sequence.

Fig. 22 shows a histogram of each realization of \( \zeta_i \) along with the estimated pdf \( p(\zeta_i) \). The pdfs were obtained by estimating the \( \mu_i \) and \( \lambda_i \) parameters associated with the Laplace pdf (18). It can be seen that the Laplace pdf consistently matched the experimental data.

VII. CONCLUSION

This paper presented an SDR architecture for cellular CDMA-based navigation. Models of the cellular CDMA signals were first developed and optimal extraction of relevant positioning and timing information was discussed. Next, a description of the acquisition and tracking stages of a LabVIEW-based SDR was presented. The statistics of the pseudorange error of the proposed SDR in an additive white Gaussian channel were derived. Furthermore, the discrepancy between the clock biases observed by a receiver in two different
sectors of the BTS cell was analyzed and modeled as a stochastic dynamic sequence. The consistency of the obtained model was experimentally analyzed in different locations, at different times, and for different cellular providers. Finally, experimental results validating the pseudoranges produced by the proposed SDR were presented, in which the SDR’s pseudoranges followed closely the true range between mobile UAV-mounted and car-mounted receivers and two cellular BTSs.

**ACKNOWLEDGMENT**

The authors would like to thank Souradeep Bhattacharya for his help in data collection.

**REFERENCES**


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