Maximizing Profit on User-Generated Content Platforms with Heterogeneous Participants

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Abstract—In this paper, we consider a user-generated content platform monetized through advertising and managed by an intermediary. To maximize the intermediary’s profit given the rational decision-making of content viewers and heterogeneous content producers, a payment scheme is proposed in which the intermediary can either tax or subsidize the content producers. First, we use a model with a representative content viewer to determine how the content viewers’ attention is allocated across available content by solving a utility maximization problem. Then, by modeling the content producers as self-interested agents making independent production decisions, we show that there exists a unique equilibrium in the content production stage, and propose a best-response dynamics to model the decision-making process. Next, we study the intermediary’s optimal payment based on decisions made by the representative content viewer and the content producers. In particular, by considering the well-known quality-adjusted Dixit-Stiglitz utility function for the representative content viewer, we derive explicitly the optimal payment maximizing the intermediary’s profit and characterize analytical conditions under which the intermediary should tax or subsidize the content producers. Finally, we generalize the analysis by considering heterogeneity in terms of production costs among the content producers.

I. INTRODUCTION

As the Internet has been penetrating every aspect of our lives, we have witnessed a significant expansion of online user-generated content platforms during the past decade. Well-known examples of such platforms include YouTube, Facebook, Twitter, and Yahoo! Answers (an online community where people share knowledge). On these platforms, users can view content for free and can post content on the platform voluntarily. In addition, advertising accounts for a major source of platforms’ revenue while platforms may subsidize or tax users who provide content.

In this paper, we consider a user-generated content platform and study the profit maximization problem of the platform owner (which we refer to as intermediary), especially focusing on the question of whether it should subsidize content producers for their contribution or tax them for using the platform’s service. Subsidizing content producers can be considered as a reward for providing content, whereas taxing content producers can be considered as a usage fee for utilizing the intermediary’s resources (e.g., storage space). There is no simple answer to the question of which strategy is better for the intermediary. One the one hand, if the intermediary subsidizes content producers, it gives away some of his advertising revenue to them, but the total advertising revenue may increase due to the increased amount of content available on the platform. On the other hand, if it taxes content producers, the intermediary obtains direct revenue from them while he may suffer from reduced revenue from advertisement by discouraging content production. In practice, “subsidizing content producers” can be observed more often (e.g., YouTube Partner, Squidoo) than “taxing content producers” (e.g., Google Picasa, which charges its users for storage exceeding the free quota).

We aim to provide a formal analysis on when subsidizing or taxing content producers is profit-maximizing. We consider a class of payment schemes in which the intermediary subsidizes or taxes content producers per content view while it provides the service for free to content viewers. Subsidizing content producers per content view is a common practice in the Internet industry (e.g., YouTube Partner), while not taxing users for viewing content is also a common practice (e.g., YouTube, Yahoo! Answers). Typically, the number of users who view content without producing content is much larger than that of users who actively produce content (see, for example, [11] for a study on the Java Forum). Hence, in practice, it is relatively easier to use a payment scheme on the side of content producers than on the side of content viewers.

In order to study the optimal payment of the intermediary, we model the strategic interactions among the intermediary, content producers and content viewers using a two-sided market [6]. We analyze a three-stage game in which the intermediary sets a payment rate, content producers decide whether to produce content or not, and content viewers allocate their content views over available content. We use backward induction to analyze the game. First, we solve the utility maximization problem of a representative content viewer in order to study the allocation of content views. Next, we study the equilibrium production decisions of content producers given a payment chosen by the intermediary and anticipating the rational decision of the representative content viewer. Lastly, we formulate and analyze the intermediary’s problem of finding an optimal payment. Throughout the paper, we use the quality-adjusted Dixit-Stiglitz utility function for the representative content viewer to illustrate our analysis. In addition, we extend our model by introducing heterogeneity in terms of production costs among content producers.

The rest of this paper is organized as follows. Related work is reviewed in Section II. Section III describes the model. In Section IV, we study the decisions made by the content viewers and content producers, and derive the optimal
TABLE I
COMPARISON BETWEEN DIFFERENT MECHANISMS

<table>
<thead>
<tr>
<th>Mechanism/Scheme</th>
<th>Purpose</th>
<th>Tax</th>
<th>Subsidize</th>
<th>Application Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eliminating low-quality content</td>
<td>Incentivize high-quality content</td>
<td>N/A</td>
<td>N/A</td>
<td>User-generated content platforms</td>
</tr>
<tr>
<td>Review-based scoring rule</td>
<td>Encourage early responders and high-quality answers</td>
<td>N/A</td>
<td>N/A</td>
<td>Q&amp;A forums</td>
</tr>
<tr>
<td>Virtual reward [3]</td>
<td>Maximize task competition probability</td>
<td>N/A</td>
<td>Users (virtual currency)</td>
<td>Social networks</td>
</tr>
<tr>
<td>Payment transfer [4]</td>
<td>Maximize social welfare</td>
<td>N/A</td>
<td>Users</td>
<td>P2P networks</td>
</tr>
<tr>
<td>Pricing content providers [8]</td>
<td>Maximize profit</td>
<td>Content providers</td>
<td>Price-sensitive users</td>
<td>Communications markets</td>
</tr>
<tr>
<td>Pricing consumers [7]</td>
<td>Maximize profit</td>
<td>Consumers</td>
<td>N/A</td>
<td>Online games &amp; dating</td>
</tr>
<tr>
<td>Proposed payment scheme</td>
<td>Maximize profit</td>
<td>Content producers</td>
<td>Content producers</td>
<td>User-generated content platforms</td>
</tr>
</tbody>
</table>

payment maximizing the intermediary’s profit. Heterogeneous production costs are studied in Section V. Finally, concluding remarks are offered in Section VI.

II. RELATED WORKS

We summarize in Table I several mechanisms closely related to ours.

If the intermediary chooses to subsidize the content producers, the proposed payment scheme is essentially an incentive mechanism. Various incentive mechanisms have been proposed recently. For instance, the authors in [1] proposed eliminating or hiding low-quality content to provide content producers with incentives to generate high-quality content. In [2], two scoring rules, i.e., the approval-voting scoring rule and the proportional-share scoring rule, were proposed to enable the most efficient outcome for online question and answer forums (e.g., Yahoo Answers). The authors in [3] proposed a (virtual) reward-based incentive mechanism to improve the overall task completion probability in collaborative social media networks. Pricing-based incentives were proposed to improve the social welfare for peer-to-peer networks in [4] and to maximize any system utility in multi-user relay networks in [5], respectively.

If the intermediary taxes the content producers, then the proposed scheme can be classified as market pricing. Moreover, user-generated content platforms can be naturally modeled as two-sided markets, where two user groups interact and provide each other with network benefits. Nevertheless, most two-sided market research neglected the intra-group externalities (e.g., see [6] for a survey), which in the contexts of user-generated content platforms indicate the content producers’ competition for content views. By considering a general two-sided market, the authors in [7] studied the tradeoffs between the merchant mode and the platform mode, and showed the conditions under which the merchant or platform mode is preferred. The authors in [8] studied the broadband communications market based on a two-sided model, and proposed pricing the content providers to maximize the service provider’s profit. A few recent studies on two-sided markets explicitly considered intra-group externalities. For instance, [9] studied the optimal pricing problem to maximize the platform’s profit for the payment card industry with competition among the merchants. More recently, considering intra-group competition, [10] studied the problem of whether an entrant platform can divert agents from the existing platform and make a profit. Nevertheless, the existing studies on two-sided markets typically neglected content heterogeneity/substitutability as well as buyers’ “love for variety”. Unlike general two-sided market research (e.g., [6]), this paper considers both the content producers’ competition for content views and the content heterogeneity/substitutability, which are key features of user-generated content platforms and, as shown in this paper, significantly impact the intermediary’s optimal payment.

III. MODEL

In this section, we specify the modeling details of the intermediary, content producers and viewers.

A. Intermediary

The advertising revenue is in general proportional to the total number of times that the content with advertisement is viewed (i.e., content views). To increase the advertising revenue, one natural approach is to increase the content views. To do so, we propose that the intermediary provides the content producers with economic incentives (i.e., subsidizing) to produce more high-quality content, which in turn attracts more content views. Essentially, this scheme allows the intermediary to share with the content producers (part of) its advertising revenue as an economic incentive, and it has been exercised by several content platforms (e.g., YouTube Partner, Squidoo). Note that, since we focus on the side of content producers, we do not consider subsidizing the content viewers (e.g., providing rewards) to attract more content views. Moreover, the number of content viewers is typically much greater than that of content producers and hence, in practice, it is relatively easier to implement the proposed payment scheme on the side of content producers.

To formally state our model, we denote $\bar{x}$ as the total content views of all the content on the intermediary’s platform, and $b \geq 0$ as the (average) advertising profit (i.e., revenue minus cost) that the intermediary can derive per content view. The intermediary pays $\theta$ per content view to the respective content producers. For the completeness of analysis, we allow $\theta$ to take negative values, in which case the intermediary taxes the content producers with $-\theta$ per content view. Practically speaking, negative $\theta$ may correspond to that the intermediary...
taxes the content producers for utilizing its resources (e.g., bandwidth) or for commission fees if the content producers produce advertisement-type content. In the following analysis, we use the term payment to refer to \( \theta \) wherever applicable, regardless of its positive or negative sign. Neglecting the intermediary’s recurring fixed operational cost, we can express the intermediary’s profit as

\[
\Pi_I = (b - \theta) \cdot \bar{x}.
\]

We assume throughout the paper that the paper that is exogenously determined and fixed, and shall focus on deriving the optimal \( \theta \) that maximizes the intermediary’s profit. Note that in our current study, we restrict our analysis to uniform and fixed, and shall focus on deriving the optimal \( \theta \) regardless of its positive or negative sign. Neglecting the payment we use the term payment or different payment for different content or content producers), which is left as our future work.

### B. Content Producers

As evidenced by the exploding number of YouTube users, a popular user-generated content platform can attract a huge number of content producers. To capture this fact, we use a continuum model and assume that the mass of content producers is normalized to one. For the convenience of analysis and focusing on the impacts of content quality on the optimal payment \( \theta \), we further assume that the content producers are differentiated by the quality of content they produce, whereas the heterogeneity in terms of other factors (e.g., production cost) is ignored. The content quality is represented by a scalar and, in practice, may be measured by a combination of the popularity and technical specifications such as video resolution if the content is video. Mathematically, we denote \( q_i \geq 0 \) and \( c > 0 \) as the quality of content produced by content producer \( i \) and the production cost, respectively.\(^1\) Without causing ambiguity, we occasionally use content \( q \), to refer to the content with a quality \( q_i \). To characterize heterogeneity in the content quality, we assume that the content quality follows a cumulative distribution function (CDF), denoted by \( F(q) \), across the unit mass of content producers. In other words, \( F(q) \) denotes the number or fraction of content producers whose content has a quality less than or equal to \( q \geq 0 \).

Millions of users engaging daily in Internet activities such as blogs, for which they receive no monetary rewards, highlight that such content producers may simply derive satisfaction (and hence utility) by attracting the content viewers’ attention\([1][3][16]\). We use the content views to quantify the amount of received attention and assume that the benefit resulting from the content viewers’ attention for a content producer is a linear function of its content views. We assume further that each content producer \( i \) is self-interested and can strategically make a binary decision: produce or not produce. Denote by \( x(q_i) \geq 0 \) the number of views for content \( q_i \). If content producer \( i \) produces content on the intermediary’s platform, it can derive a utility expressed as

\[
\pi_i = x(q_i) \cdot (\theta + s) - c,
\]

where \( s \geq 0 \) is the benefit per content view derived from the content viewers’ attention,\(^2\) \( \theta \) is the payment per content view determined by the intermediary, and \( c \) is the production cost. Content producer \( i \) obtains zero utility if it chooses not to produce content. By the assumption of rationality, content producer \( i \) chooses to produce content if and only if its utility is non-negative.\(^3\)

In what follows, we assume that the content quality \( q \) follows a distribution in a normalized interval \([0, 1]\) and the probability density distribution (PDF) is given by a continuous and positive function \( f(q) \) for \( q \in [0, 1]\). Scaling the interval \([0, 1]\) to \([0, \bar{q}]\) does not affect the analysis, but will only complicate the notations. It is intuitively expected that a content with a higher quality will attract more content views (and yield a higher utility for its content producer, too) than the one with a lower quality. Thus, the production decision of the content producers has a threshold structure. In particular, there exist marginal content producers whose content has a quality of \( q_m \), and those content producers whose content quality is greater (less) than \( q_m \) will (not) choose to produce content on the intermediary’s platform. We refer to \( q_m \) as the marginal content quality. Next, it is worthwhile to provide the following remarks concerning the model of content producers.

**Remark 1:** In our model, a content producer who produces \( m \geq 1 \) pieces of content is viewed as \( m \) content producers, each of whom produces a single content, and the total production cost is \( m \cdot c \) (i.e., constant returns to scale \([7]\)).

**Remark 2:** Focusing on the content quality heterogeneity, our model does not take into account the content producers’ decisions regarding their content quality. Instead, like in \([7]\), we assume that the content producers will incur a predetermined production cost if they choose to produce content. For the ease of presentation and to gain insights as to whether the intermediary should tax or subsidize the content producers, we first consider a homogeneous production cost among the content producers. In Section V, we shall generalize the model to consider heterogeneity in the content producers’ production costs.

**Remark 3:** If \( \theta < -s \), it is clear from (2) that no content producers can possibly receive a non-negative utility by producing content on the platform. As a consequence, \( \bar{x} = 0 \) and the intermediary’s profit is zero. On the other hand, if \( \theta > b \), then we see from (1) that the intermediary can never gain a positive profit. Hence, we exclude these two trivial cases in the remainder of this paper and focus on \(-s \leq \theta \leq b\) unless otherwise stated.

### C. Content Viewers

Despite that the content viewers are diverse in terms of preferences towards the content, the aggregate content viewing decisions of all the content viewers can be conveniently characterized by the decision of a representative content viewer.

\(^1\)We assume that each content producer knows or has a accurate estimate of its own content quality and production cost before producing content (e.g., based on its past experiences).

\(^2\)Essentially, \( s \geq 0 \) converts the content viewer’s attention to the content producers’ (economic) benefit/utility.

\(^3\)Specifying an alternative tie-breaking rule (e.g., random choice between producing and not producing) in case of \( \pi_i = 0 \) will not affect the analysis of this paper, since the fraction of indifferent content producers is zero under the continuum model.
Thus, we adopt the widely-used representative agent model to determine how the total content views are allocated across the content [14]. Specifically, subject to a content view constraint specified by $T > 0$ (which can be interpreted as the total market size), the representative content viewer optimally allocates its content views across the available content to maximize its utility. On the Internet, it is quite common that multiple content platforms offer similar services and the content viewers have access to the content on any of these platforms. Focusing on the intermediary’s optimal payment decision, we do not consider the details of how the content is produced on the other platforms. Instead, we can assume that the mass of content available on the other platforms is $n_a \geq 0$ and the content quality follows a certain CDF $F(q)$ with support $q \in [q_l, q_h]$, where $0 \leq q < q_h$ are the lowest and highest content quality on the other platforms, respectively. For the convenience of notation, throughout the paper, we alternatively represent the content on the other platforms using a unit mass of content with an aggregate quality of $q_a$, without affecting the analysis. Note that $q_a$ is a function of $n_a \geq 0$, $F(q)$ and the utility function of the representative content viewer. In particular, given a uniform distribution of content quality on the other platforms and the quality-adjusted Dixit-Stiglitz utility for the representative content viewer (which we shall define later), we can readily obtain

$$q_a = \left[ n_a \left( q_h^{+1} - q_l^{+1} \right) \right]^{\frac{1}{\sigma}}, \quad (3)$$

where $\sigma > 1$ measures the content substitutability. Recalling that $q_m \in [0, 1]$ is the marginal content quality above which the content producers choose to produce content on the intermediary’s platform, we write the representative content viewer’s utility function as $U(x(q), x_a | q_m, q_a)$, where $x(q)$ denotes the content view for content $q \in [q_m, 1]$ and $x_a$ is the content view allocated to the content $q_a$ on the other content platforms. Note that $x(q)$ can be rewritten as $x(q | q_m, q_a)$, although we use the succinct notation $x(q)$ throughout the paper whenever applicable. If $q_m$ increases (decreases), there will be less (more) content on the intermediary’s platform. Because of the continuum model, we allow $x(q)$ and $x_a$ to take non-integer values, and $x(q)$ actually represents the content view density allocated to a continuum of content with quality $q \in [q_m, 1]$. Next, we formulate the utility maximization problem for the representative content viewer as follows

$$\max_{x(q) \geq 0, x_a \geq 0} U(x(q), x_a | q_m, q_a),$$

t.s.t., $\int_{q_m}^{1} x(q)dF(q) + x_a \leq T, \quad (4)$

where $F(q)$ is the CDF of content quality on the intermediary’s content platform. It is worth noting that an implicit assumption underlying the problem (4) is that the aggregate quality of the content on the other platforms is independent of the intermediary’s payment decision. This can be justified by noting that there are many content platforms on the Internet and changes on one content platform have a negligible impact on the other platforms. Before performing further analysis, we assume that the following properties are satisfied by the utility function $U(x(q), x_a | q_m, q_a)$.

**Property 1 (Diminishing returns):** $U(x(q), x_a | q_m, q_a)$ is increasing and strictly concave in $x(q)$ and $x_a$, for $q \in [0, 1]$.  

**Property 2 (Preference towards diversified bundle of content):** $\max_{x(q) \geq 0, x_a \geq 0} U(x(q), x_a | q_m, q_a)$ is decreasing in $q_m \in [0, 1]$.  

**Property 3 (Negative externalities):** Denote by $x^*(q | q_m, q_a)$, for $q \in [0, 1]$, the optimal solution to (4). If content $q$ is produced, then $x^*(q | q_m, q_a)$ is positive. Moreover, it is continuous and strictly increasing in $q_m \in [0, 1]$, increasing in $q \in [0, 1]$, and decreasing in $q_a$ for $q_a \in [0, \infty)$. In particular, $x^*(0 | q_m, q_a) = 0$ for all $q_m \in [0, 1]$ and $q_a \geq 0$.  

**Property 4 (More content leading to more content views):** $x = \int_{q_m}^{1} x^*(q | q_m, q_a)dF(q)$ is decreasing in $q_m \in [0, 1]$.  

We briefly discuss the above properties. Property 1 captures the effects of diminishing returns when the representative content viewer views more content. Property 2 models the phenomenon that content viewers will typically benefit from the participation of content producers on the platform. This is particularly true for online content platforms, where the content viewers prefer to view a diversified bundle of content. Thus, when $q_m \in [0, 1]$ increases, i.e., fewer content producers produce content, the representative content viewer’s (maximum) utility decreases. Property 3 reflects the “crowding effects”, i.e., there exist (indirect) negative network externalities among the competing content producers. Specifically, an individual content producer will attract a smaller number of content views if more content producers choose to produce content on the platform or the aggregate content quality on the other platforms is higher. The last property ensures that more content views are devoted to the intermediary’s platform if there is more content available on the platform.  

As a concrete example satisfying Properties 1–4, we use a *quality-adjusted* version of the well-known Dixit-Stiglitz utility which is a classic utility function capturing the content substitutability and the content producers’ competition for content views [13][14], defined as below$^5$

$$U(x(q), x_a | q_m, q_a) = \left[ \int_{q_m}^{1} q \cdot |x(q)|^{\frac{1}{\sigma}} dF(q) + q_a \cdot x_a^{\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}}, \quad (5)$$

where $\sigma > 1$ measures the elasticity of substitution between different content. In the extreme case, the content is perfectly substitutable when $\sigma = \infty$ [13].

$^4$Due to the continuum model, $\hat{x}(q | q_m, q_a) = x^*(q | q_m, q_a)$ almost everywhere for $q \in [0, 1]$ is also optimal in maximizing the utility function. For simplicity, we treat such $x(q | q_m, q_a)$ the same as $x^*(q | q_m, q_a).$ The treatment does not change our analysis, except that it affects the decisions of a negligible mass of content producers.

$^5$If we model the quality of $n_a$ pieces of content on the other platforms by using a certain CDF $F(q)$ for $q \in [q_l, q_h]$, where $0 \leq q < q_h$ are the lowest and highest content quality on the other platforms, respectively, then the Dixit-Stiglitz utility function in (5) becomes $U(x(q), x_a | q_m, q_h) = \int_{q_m}^{1} q \cdot |x(q)|^{\frac{1}{\sigma}} dF(q) + n_a \int_{q_l}^{q_h} q \cdot |x(q)|^{\frac{1}{\sigma}} dF(q)$ and the analysis remains the same.
IV. PROFIT MAXIMIZATION ON CONTENT PLATFORMS

In this section, based on the model described above and the proposed payment scheme, we study the problem of optimizing payment in the presence of self-interested content producers and content viewers. The timing can be described as follows.

Stage 1 (Pricing Decision): The intermediary announces the value of $\theta$ to the content producers.

Stage 2 (Production Decision): Given $\theta$, each content producer makes a binary decision as to whether or not to produce content on the platform.

Stage 3 (Viewing Decision): Given the available content, the content viewers, consolidated as a representative content viewer, optimally allocate the content views to maximize the utility subject to a total content view constraint.

From the described timing, we see that the intermediary can be regarded as the leader, followed by the content producers and then by the content viewers. In the following, we proceed with our analysis using backward induction.

A. Optimal Content Viewing

It follows from the strict concavity of $U(x(q), x_a | q_m, q_a)$, specified in Property 1, that there exists a unique optimal solution, denoted by $x^*(q)$ and $x^*_a$ for $q \in [0, 1]$, to the utility maximization problem in (4), although it is not possible to obtain a closed-form expression without specifying $U(x(q), x_a | q_m, q_a)$. By considering the quality-adjusted Dixit-Stiglitz utility defined in (5) and uniform distribution of the content quality, we can obtain explicitly the closed-form solution as follows

$$x^*(q) = \frac{T(\sigma + 1)q^\sigma}{(\sigma + 1) q^\sigma + (1 - q_m^{\sigma + 1})},$$

for $q \in [q_m^{-1}, 1]$, $x^*(q) = 0$ for $q \in [0, q_m)$ and $x^*_a = \frac{T(\sigma + 1)q_a^\sigma}{(\sigma + 1) q_a^\sigma + (1 - q_m^{\sigma + 1})}$. The details of deriving (6) are omitted for brevity. After plugging $x^*(q)$ and $x^*_a$ into (5), the maximum utility derived by the representative content viewer is given by

$$U^*(x^*(q), x^*_a) = T \left[ q_m^\sigma + \frac{1 - q_m^{\sigma + 1}}{\sigma + 1} \right],$$

which is decreasing in $q_m \in [0, 1]$.

B. Equilibrium Content Production

Due to rationality, content producers will not choose to produce content if they cannot obtain non-negative utilities. Essentially, the production decision can be formalized as a non-cooperative game with an infinite number of players, the solution to which is (Nash) equilibrium. At an equilibrium, if any, no content producers can gain more benefits by deviating from their decisions. In other words, the fraction of content producers choosing to produce content on the intermediary’s platform does not change at the equilibrium, or equivalently, the marginal content quality $q_m \in [0, 1]$ becomes invariant.

Next, we study the equilibrium content production by specifying the equilibrium marginal content quality denoted by $q^*_m$.

If $q^*_m = 1$, then no (or a zero mass of) content producers can receive a non-negative utility by producing content on the platform. This implies that, with $q^*_m = 1$, we have $x^*(1 | q_m, \theta) \cdot (\theta + s) - c \leq 0$. If there are some content producers choosing to produce content at the equilibrium (i.e., $q^*_m \in (0, 1)$), then according to the definition of marginal content producers, we have $x^*(q^*_m | q_m, q_a) \cdot (\theta + s) - c = 0$. Hence, we can show that $q^*_m \in [0, 1]$ satisfies

$$q^*_m = Q(q^*_m) = \left[ \arg\min_q \left\{ x^*(q | q_m, q_a) \cdot (\theta + s) - c \geq 0 \right\} \right]_0^1,$$

(8)

where $[x]_0^1 = \min\{1, \max\{0, x\}\}$. Thus, an equilibrium point of content production exists if and only if the mapping $Q(q^*_m)$, defined in (8), has a fixed point. Next, we formally define the equilibrium marginal content quality in terms of $q^*_m$ as below.

Definition 1: $q^*_m$ is an equilibrium marginal content quality if it satisfies $q^*_m = Q(q^*_m)$.

We establish the existence and uniqueness of an equilibrium marginal content quality in Theorem 1, whose proof can be found in [17].

Theorem 1. For any $\theta \in [-s, b]$, there exists a unique equilibrium $q^*_m \in (0, 1)$ in the production decision stage. Moreover, $q^*_m$ satisfies

$$\begin{cases} q^*_m = 1, & \text{if } x^*(1 | q_m, \theta) \cdot (\theta + s) \leq c, \\ q^*_m \in (0, 1), & \text{otherwise,} \end{cases}$$

(9)

where $x^*(1 | q_m, \theta)$ is obtained by solving (4) with $q_m = 1$. □

Theorem 1 guarantees the existence of a unique equilibrium point and shows that if the the content producer with the highest quality cannot obtain a positive utility (due to high production cost, taxing or low subsidizing from the intermediary), then no content producers choose to produce content on the intermediary’s content platform at the equilibrium. For notational convenience, we denote the value of $\theta$ that satisfies $x^*(1 | q_m, \theta) \cdot (\theta + s) = c$ by

$$\theta = \frac{c}{x^*(1 | q_m, \theta) - s}.$$

Then, it follows from Theorem 1 that the intermediary can gain a positive profit if and only if $\theta \in (\bar{\theta}, b]$. Nevertheless, if $\theta \geq b$, then the intermediary’s profit is always zero. Hence, we assume $\theta < b$ throughout the paper. Based on the uniqueness of $q^*_m$ for any $\theta \in [-s, b]$, we can express $q^*_m = Q_m^*(\theta)$ as a function of $\theta \in [-s, b]$. While there exists no simple closed-form expression of $q_m^*(\theta)$ in general, it can be easily shown that $q^*_m(\theta) \in (0, 1)$ is strictly decreasing in $\theta \in (\bar{\theta}, b]$ and $q^*_m(\theta) = 1$ for $\theta \in [-s, \bar{\theta}]$. In practice, the content producers do not have complete information regarding each other and hence, they may not make directly the decisions that strikes an equilibrium. Instead, an adjustment process where the content producers update their decisions based on limited information is required.

A natural and well-studied approach to modeling an adjustment process is the best-response dynamics, in which each
decision maker chooses the best action in response to the decisions made by the others. In this paper, we consider the best-response dynamics based on naive (or static) expectation. Specifically, at the end of time $t=1,2,3,\cdots$, content producer $i$ assumes that the decisions made by the other content producers at time $t+1$ remain the same as those at time $t$, and expects $x_{t+1}(q_i) = x^*(q_{m,t},q_a)$, where $x^*(q_{m,t},q_a)$ is the solution to (4) and $q_{m,t} \in [0,1]$ is the marginal content quality at time $t$. Note that a content producer with a content quality less than $q_{m,t}$ may also choose to produce content at time $t+1$, if it believes that there is not much high quality content on the platform (i.e., $q_{m,t} \in [0,1]$ is large) and it can receive a non-negative utility. Similar decision processes have been adopted in the existing literature (e.g., [12] and references therein). The best-response decision model implies that the sequence $q_{m,t}$, for $t=0,1,2,\cdots$, evolves as follows: 

$$q_{m,t+1} = Q(q_{m,t}),$$

(11)

where $Q(\cdot)$ is defined in (8). Essentially, the dynamics in (11) is a fixed point iteration for $Q(\cdot)$ and it converges regardless of the initial point if $|Q'(q)| < 1$ for $q \in [0,1]$ [15]. It should be noted that, by considering the dynamics specified by (11), we implicitly assume that the content produced in the previous periods has little value and will not significantly affect the content views in the current period (e.g., news content platform). Moreover, the dynamics specified by (11) requires that all the content producers update production decisions at the end of each time period. In practice, if only a fraction $\epsilon \in (0,1]$ of the content producers make decisions each time, then the sequence becomes $q_{m,t+1} = (1-\epsilon)q_{m,t} + \epsilon Q(q_{m,t})$ without affecting the equilibrium analysis.

Now, we show more specific results by applying the equilibrium analysis to the example of Dixit-Stiglitz utility and uniform distribution of content quality. In particular, given the representative content viewer’s utility function defined in (5), we can show that $q^*_m$ satisfies

$$q^*_m = Q(q^*_m) = \left\{ \frac{c \left[ (\sigma + 1) \cdot q^*_a + 1 - (q^*_m)^{\sigma+1} \right]}{T(\sigma + 1)(\theta + s)} \right\}^{\frac{1}{\sigma}}.$$

(12)

The considered best-response dynamics $q_{m,t}$, for $t=0,1,2,\cdots$, is specified as $q_{m,t+1} = Q(q_{m,t})$, where $Q(\cdot)$ is given by (12). Based on Theorem 1, the existence and uniqueness of equilibrium point is readily available. Moreover, we show in the following proposition a sufficient condition for the convergence of the considered best-response dynamics $q_{m,t+1} = Q(q_{m,t})$.

**Proposition 1.** Suppose that $U(x(q),x_a | q_m,q_a)$ is given by the Dixit-Stiglitz utility function in (5) and the content quality $q$ is uniformly distributed in $[0,1]$. For any $\theta \in [-s,b]$, the sequence $q_{m,t+1} = Q(q_{m,t})$, where $Q(q_{m,t})$ is specified by (12), converges to the unique equilibrium $q^*_m$ starting from any points $q_{m,0} \in [0,1]$ if $q^*_m < \sigma$.

Proof: The proof is available in [17]. □

The intuition underlying the condition $q^*_m < \sigma$ is that, if $q_a$ is too large and some content producers choose to produce content at time $t$, then probably no content producers will choose to produce content at time $t+1$. Next, some content producers may choose to produce content again at time $t+2$ and this process may repeat without convergence. Note that $q^*_m < \sigma$ is only a sufficient condition for convergence of the best-response dynamics. Even though the condition $q^*_m < \sigma$ is violated, we observe through numerical results that the best-response dynamics always reach the unique equilibrium point $q^*_m = 1$ in one step regardless of the initial points.

**C. Optimal Price**

Based on decisions made by the content viewers and content producers, we study the optimal payment $\theta$ that maximizes the intermediary’s steady-state profit (i.e., profit obtained when the content production decision stage reaches the equilibrium). Mathematically, we formalize the profit maximization problem as

$$\theta^* = \max_{\theta \in [\underline{\theta}, \bar{\theta}]} (b - \theta) \cdot \bar{x},$$

(13)

where $\bar{x} = \int_{q_m^*}^{1} x^*(q) dF(q)$. The decision interval is shrunk to $[\underline{\theta}, \bar{\theta}]$, since $\theta \in [-s, \theta^*]$ always results in a zero profit for the intermediary, where $\underline{\theta}$ is defined in (10). By Property 4 stated in Section III-C, $\bar{x} = \int_{q_m^*}^{1} x^*(q) dF(q)$ is decreasing in $q_m^* \in [0,1]$. Then, recalling that $q_m^*(\theta)$ is strictly decreasing in $\theta \in [\underline{\theta}, \bar{\theta}]$, we can see $\bar{x}$ is increasing in $\theta \in [\underline{\theta}, \bar{\theta}]$. Although we can numerically solve the profit maximization problem (13), it is rather challenging, if not impossible, to explicitly determine the optimal payment $\theta^*$ without specifying the utility function $U(x(q),x_a | q_m,q_a)$ or further restrictions on $\bar{x}$. If the profit function in (1) is strictly concave in $\theta$, then there exists a unique optimal payment $\theta^* \in [\underline{\theta}, \bar{\theta}]$ maximizing the intermediary’s profit and satisfying the first-order optimality condition

$$-\bar{x}(\theta^*) + (b - \theta^*) \frac{\partial \bar{x}}{\partial \theta} \bigg|_{\theta=\theta^*} = 0.$$

(14)

Moreover, if the first-order partial derivative of (1) with respect to $\theta = 0$ is negative (positive), then the optimal payment $\theta^*$ is negative (positive), i.e., the intermediary should tax (subsidize) the content producers to maximize its profit. In the following analysis, to gain insights and explicitly derive the optimal payment $\theta^*$, we consider the quality-adjusted Dixit-Stiglitz utility and uniform distribution of content quality. A closed-form optimal payment $\theta^* \in [\underline{\theta}, \bar{\theta}]$ is explicitly obtained and shown in Theorem 2.

**Theorem 2.** Suppose that $U(x(q),x_a | q_m,q_a)$ is given by the Dixit-Stiglitz utility function in (5) and the content quality $q$ is uniformly distributed in $[0,1]$. The unique optimal payment $\theta^* \in [\underline{\theta}, \bar{\theta}]$ that maximizes the intermediary’s profit is given by

$$\theta^* = \frac{c \left[ (\sigma + 1) \cdot q^*_a + 1 - z^{\sigma+1} \right]}{T(\sigma + 1) \cdot z^\sigma} - s,$$

(15)
where $z \in [q_m^*, b, 1]$ is the unique root of the equation

$$T \cdot q_a^* \cdot (b + s) - \frac{c}{(\sigma + 1) \cdot q_a^* + 1 - z^{\sigma + 1} + 1} \cdot z^{\sigma + 1} = 0.$$  \hspace{1cm} (16)

Proof: See [17]. \hfill \Box

Having derived the optimal payment $\theta^*$, we next analyze the sign of the optimal payment in Proposition 2. Such analysis is useful in understanding the impacts of various factors on the intermediary’s decision of taxing or subsidizing.

**Proposition 2.** Suppose that $U(x(q), x_a | q_m, q_a)$ is given by the Dixit-Stiglitz utility function in (5) and the content quality $q$ is uniformly distributed in $[0, 1]$. The optimal payment $\theta^* \in \left[\frac{c \cdot q_a^*}{T} - s, b\right]$ that maximizes the intermediary’s profit satisfies

$$\begin{cases}
\theta^* \in (0, b), & \text{if } \Delta < \frac{c(\sigma + 1) \cdot q_a^* \cdot (b + s)}{s^2 T}, \\
\theta^* = 0, & \text{if } \Delta = \frac{c(\sigma + 1) \cdot q_a^* \cdot (b + s)}{s^2 T}, \\
\theta^* \in \left(\frac{c \cdot q_a^*}{T} - s, 0\right), & \text{if } \Delta > \frac{c(\sigma + 1) \cdot q_a^* \cdot (b + s)}{s^2 T},
\end{cases}$$

where $\Delta = \frac{\sigma}{q_a^*(0)} + [q_m^*(0)]^\sigma$, in which $q_m^*(0)$ is the equilibrium point of content production when the intermediary chooses $\theta = 0$.

Proof: The proof is given in [17]. \hfill \Box

Proposition 2 rigorously characterizes the conditions for subsidizing or taxing. Meanwhile, it also enables us to obtain some qualitative results regarding whether taxing or subsidizing is the optimal choice to maximize the intermediary’s profit. Specifically, subsidizing should be selected if one of the following cases is satisfied:

1. Total content view $T$ (i.e., market size) is sufficiently small;
2. Production cost $c$ is sufficiently large;
3. Benefit per content view $s$ is sufficiently small;
4. Aggregate content quality on the other platforms $q_{a}$ is sufficiently large;
5. Advertising revenue per content view $b$ is sufficiently large.

In the first four cases, few content producers can receive a non-negative utility by producing content without being subsidized by the intermediary (e.g., if the production cost $c$ is very high, then content producers need to receive subsidiary from the intermediary to cover part of the production cost). As a result, the intermediary cannot attract enough content views or maximize its profit without subsidizing the content producers. The last case indicates that if the intermediary can derive a sufficiently high advertising revenue per content view, then it can share the advertising revenue with the content producers to encourage more content production. Numerical results illustrating the impacts of $q_m$, $c$ and $T$ are shown in [17].

Finally, we conclude this section by discussing two extreme cases, $q_a \to 0$ and $\sigma \to \infty$. When $q_a \to 0$, the aggregate content quality on the other platforms is negligible (e.g., very low quality or little content available). In other words, the intermediary becomes a monopolist in the market, and almost all the content views are devoted to the content on the intermediary’s platform. Therefore, the intermediary can tax the content producers by choosing $\theta^* \to -s$ and its profit can be arbitrarily close to $(b + s)T$. When $\sigma \to \infty$, the content becomes perfectly substitutable. Naturally, all the content views will be attracted by the content with the highest quality. This can also be verified by taking the limit $\sigma \to \infty$ in (6). Therefore, if $q_a > 1.8$ then the content produced on the intermediary’s platform will receive no content views and the intermediary cannot possibly obtain a positive profit by varying $\theta$. On the other hand, if $q_a < 1$ (which is equivalent to $q_b < 1$ when $\sigma \to \infty$), then the content with a quality of 1 can receive almost all the content views and the intermediary can set $\theta^* \to -s$ to make its profit arbitrarily close to $(b + s)T$.

To sum up, when $q_a \to 0$ or $\sigma \to \infty$ with $q_a < 1$, the intermediary can almost fully extract two sources of revenues, i.e., advertising and content producers.

V. Extension to Heterogeneous Production Costs

We generalize the preceding analysis by considering heterogeneous production costs. Due to the space limitation, we only show the main results without providing proofs. Interested readers may refer to [17] for a complete version of this section.

To keep the analysis tractable, we assume that there are $K \geq 1$ possible values for content production costs, denoted by $0 < c_1 \leq c_2 \leq \cdots \leq c_K$, and refer to content producers with a production cost of $c_k$ as type-$k$ content producers. Under the continuum model, the (normalized) mass of type-$k$ content producers is $n_k > 0$ such that $\sum_{k=1}^{K} n_k = 1$. To model the content quality heterogeneity, we assume that the content quality of type-$k$ content producers follows a continuous and positive PDF denoted by $f_k(q) > 0$ for $q \in [0, 1]$ and the corresponding CDF is $F_k(q)$ for $q \in [0, 1]$. Thus, the mass of type-$k$ content producers whose content quality is less than or equal to $q \in [0, 1]$ is given by $n_k F_k(q)$. As in the case of homogeneous production cost, for type-$k$ content producers, there exists marginal content quality, denoted by $q_m^k \in [0, 1]$, and a type-$k$ content producer with content quality greater (less) than $q_m^k$ will choose (not) to produce content on the intermediary’s platform. For notational convenience, we use the vector expression $q_m = [q_{m_1}, q_{m_2}, \cdots, q_{m_K}]$ wherever applicable.

A. Optimal Content Viewing

With heterogeneous production costs, we define the strictly concave utility function for the representative content viewer as $U(x(q), x_a | q_m, q_a)$, where $x(q)$ denotes the content view for content $q \in [0, 1]$. The four properties specified in Section III-C can be similarly restated for the utility function $U(x(q), x_a | q_m, q_a)$, and are omitted in the paper for brevity. The quality-adjusted Dixit-Stiglitz utility function in (5) be-

\textit{Footnote:} It can be easily shown from (3) that $q_a > 1$ if and only if the highest content quality $q_b > 1$. 

The equilibrium marginal content quality satisfies

$$x^*(q) = \frac{Tq^\theta}{q_a^\theta + \sum_{k=1}^K \int_{q_{mk}}^{1} n_k \cdot q^\theta dF_k(q) + a^\theta \cdot x_a}$$

for $q \in [\min\{q_{m1}, q_{m2}, \ldots, q_{mk}\}, 1]$, and $x^*(q) = 0$ otherwise.

### B. Equilibrium Content Production

Following the analysis in Section IV-B for homogeneous production cost, we first formally define the equilibrium marginal content quality, denoted by $q_{m}^*$, as follows.

**Definition 2:** $q_{m}^*$ is an equilibrium marginal content quality if it satisfies $q_{m}^* = Q(q_{m})$, where $Q(q_{m}) = [Q_1(q_{m1}), Q_2(q_{m2}) \cdots Q_K(q_{mk})]$ is given by

$$Q_k(q_{m}^*) = \left[\arg\min_q \{x^*(q \mid q_{m}^*, q_a) \cdot (\theta + s) - c_k \geq 0\}\right],$$

for $k = 1, 2, \cdots K$.

It can be easily shown that $Q(q_{m}^*)$ is continuous in the compact convex set $q_{m}^* \in [0, 1]^K$. We first show two properties satisfied by the equilibrium point in the content production stage.

**Lemma 1.** The equilibrium marginal content quality satisfies $0 < q_{m1}^* \leq q_{m2}^* \cdots \leq q_{mk}^* < 1$.

**Lemma 2.** Let $k^* = \max\{k = 1, 2, \cdots K \mid q_{mk}^* < 1\}$. The following relation is satisfied at the equilibrium

$$\frac{x^*(q_{mi})}{x^*(q_{mj})} = \frac{c_{i}}{c_{j}}$$

for $i, j \in \{1, 2, \cdots k^*\}$.

Based on Lemma 1 and Lemma 2, we obtain the following theorem regarding the existence and uniqueness of the equilibrium point $q_{m}^* \in (0, 1]^K$.

**Theorem 3.** For any $\theta \in [-s, b]$, there exists a unique equilibrium $q_{m}^* \in (0, 1]^K$ in the production decision stage. Moreover, $q_{m}^*$ satisfies

$$q_{mk}^* = 1, \quad \text{if } x^*(1 \mid q_{mk}, q_a) \cdot (\theta + s) \leq c_k,$$

$$q_{mk}^* \in (0, 1), \quad \text{otherwise},$$

for $k = 1, 2, \cdots K$, where $x^*(1 \mid q_{mk}, q_a)$ is obtained by maximizing $U(x(q), x_a \mid q_{m}, q_a)$ subject to

$$\sum_{k=1}^K \int_{q_{mk}}^{1} n_k \cdot x(q) dF_k(q) + x_a \leq T$$

and $q_{mk} = [q_{m1}, q_{m2}, \cdots, q_{mk-1}, 1, 1, \cdots 1]$ satisfies (21).

Because of the crowding effects (i.e., negative network externalities, as specified in Property 4 in Section III-C), we can easily show that $x^*(1 \mid q_{mk}^*, q_a) \geq x^*(1 \mid q_{mk}^{*1}, q_a)$, for $1 \leq i \leq j \leq K$. We can also see from (22) in Theorem 3 that type-$k$ content producers will choose to produce content at the equilibrium if and only if $\theta$ is sufficiently large such that $x^*(1 \mid q_{mk}, q_a) \cdot (\theta + s) > c_k$, for $k = 1, 2, \cdots K$. For notational convenience, we define

$$\Theta = [\theta_0, \theta_1, \cdots, \theta_K, \theta_{K+1}],$$

where $-s = \theta_0 \leq \theta_1 \leq \theta_2 \cdots \leq \theta_K \leq \theta_{K+1} = b$ and

$$\theta_k = \frac{x^*(1 \mid q_{mk}, q_a) - s}{x^*(1 \mid q_{mk}, q_a) \cdot (\theta + s) - c_k}, \quad \text{for } k = 1, 2, \cdots K.$$
max \quad q^*_{mk} \in [q^*_{mk}(\theta_{k+1}), 1] \left( b + s \frac{c_k \left( q^*_{mk} + \sum_{j=1}^{k} n_j \frac{1-(q^*_{mk})^{\sigma+1}}{\sigma+1} \right)}{T(q^*_{mk})^{\sigma}} \right) \cdot \bar{x}

(24)

content production stage. If we express \( q^*_{mk} \) as a function of \( \theta \in [\theta_k, \theta_{k+1}] \), then \( q^*_{mk}(\theta) \) is decreasing in \( \theta \in [\theta_k, \theta_{k+1}] \) and \( q^*_{mk}(\theta) \in [q^*_{mk}(\theta_{k+1}), 1] \). Since there exists no simple expression of \( q^*_{mk}(\theta) \), it is rather difficult to optimize \( (b-\theta) \cdot \bar{x} \) by directly choosing the optimal \( \theta^* \). Following the proof technique in Theorem 2 [17], we can show that the profit maximization problem with heterogeneous production costs can be reformulated as (24), where \( q^*_{mk} = q^*_{mk} \cdot \left( \frac{c_k}{c_k} \right)^{\frac{1}{\sigma}} \) and 

\[ \bar{x} = T \cdot \left( 1 - \frac{q^*_{mk}}{q^*_{mk} + \sum_{i=1}^{k} \frac{n_i \cdot (1-q^*_{mk})^{\sigma+1}}{\sigma+1}} \right). \]

By showing the second-order derivative of (24) with respect to \( q^*_{mk} \in [q^*_{mk}(\theta_{k+1}), 1] \) is strictly negative, we prove that the optimization problem in (24) is strictly concave in \( q^*_{mk} \in [q^*_{mk}(\theta_{k+1}), 1] \). Thus, the unique optimal solution to (24) can be efficiently obtained. After solving (24), we can obtain the optimal payment as

\[ \theta^* = \left[ \frac{c_k \left( q^*_{mk} + \sum_{j=1}^{k} n_j \frac{1-(q^*_{mk})^{\sigma+1}}{\sigma+1} \right)}{T(q^*_{mk})^{\sigma}} \right] - s. \]

Next, based on the optimal solution to (24), we develop a recursive algorithm to find the optimal payment maximizing the intermediary’s profit and describe it in Algorithm V-C.

### Algorithm 1

Find \( \theta^* \in [-s, b] \)

\[ \Pi_T \leftarrow 0, \theta^* \leftarrow -s, \text{ and } k \leftarrow 1 \]

while \( k \leq K \)

\[ \text{Solve (24) and denote the maximum value by temp} \]

if \( \Pi_T < \text{temp} \)

\[ \Pi_T \leftarrow \text{temp} \]

\[ \theta^* = \left[ \frac{c_k \left( q^*_{mk} + \sum_{j=1}^{k} n_j \frac{1-(q^*_{mk})^{\sigma+1}}{\sigma+1} \right)}{T(q^*_{mk})^{\sigma}} \right] - s \]

end if

\[ k + + \]

end while

return \( \theta^* \)

As in the case of homogeneous production cost, we can also analyze whether the intermediary should tax or subsidize the content producers. Nevertheless, we omit the result because of its similarity with Proposition 2.

### VI. CONCLUSION

In this paper, we studied a user-generated content platform and proposed a payment scheme in which the intermediary can either tax or subsidize the content producers to maximize its profit. We first used the representative content viewer model to determine how the content viewers’ attention is allocated across different content. Then, we showed that there always exists a unique equilibrium point at which no content producers can gain more benefits, and that, under certain conditions, the equilibrium point is guaranteed to be reached through an iterative process in which the content producers update their decisions with limited information. Next, we formalized the intermediary’s profit maximization problem and, by using the quality-adjusted Dixit-Stiglitz utility function as a concrete example, derived the closed-form optimal solution explicitly. We also showed the analytical conditions under which the intermediary should tax or subsidize the content producers. Then, we discussed qualitatively the impacts of the aggregate content quality on the other platforms and content substitutability on the intermediary’s profit. Finally, we generalized our model by considering heterogeneity in the content producers’ production costs.

### REFERENCES


