Solving Payment Cost Co-optimization Problems

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***Abstract*— Current U.S. electricity markets select supply bids by using a bid cost minimization (BCM) auction mechanism but then settle the payments based on locational marginal prices (LMPs). The resulting payments can be significantly higher than the minimized bid costs. An alternative payment cost minimization (PCM) mechanism aiming to minimize the total payments has been discussed. Studies on single product problems have shown that PCM leads to reduced payments, but few results have been reported for the co-optimization of energy and other products. In view that co-optimization leads to a more efficient capacity allocation than optimizing each product individually, it is important to investigate the PCM co-optimization problems, and solve them in standard MIP solvers for a fair comparison with BCM. In PCM, prices are decision variables and need to be appropriately defined. We characterized marginal price-setting units by using logical constraints and converted them to linear forms since linearity is required by the standard MIP solvers. The nonlinear cross-product in PCM objective function, however, cannot be converted to linear forms. Based on our recent results on surrogate optimization, a method is developed to deal with nonlinearity. Prices are first fixed at their values at the previous iteration to obtain linear formulation, and are then updated using price definition if the surrogate condition is satisfied. Numerical testing results of small examples and a 24-bus example demonstrate the effectiveness and efficiency of the method.**

***Index Terms* — Branch-and-cut, co-optimization, Lagrangian Relaxation, MIP, payment cost minimization, price definition, surrogate optimization.**

I. INTRODUCTION

The Independent System Operators (ISOs) in current U.S. electricity markets use a "bid cost minimization" or BCM auction mechanism to select supply bids, but then clear the markets based on the locational marginal prices (LMPs). This bid cost minimization mechanism maximizes social welfare if supply bids are consistent with their true production costs. However, it is possible for some bidders to strategically deviate from bidding their true costs to gain more profits, thus the current auction mechanism has issues. The bid cost minimization leads to inconsistency between the auction and settlement mechanisms, and the payments can be significantly higher than the minimized bid costs. An alternative "payment cost minimization" or PCM mechanism which minimizes the total payments directly has been discussed. Illustrative examples [1] have shown that PCM leads to reduced payments for given set of bids as compared to BCM. Since bidders may bid differently under two mechanisms, a game theoretic model was used to demonstrate the consumer savings in PCM. The "ice hockey" bidding behavior was also less likely to occur in PCM [2]. However, many other aspects of the mechanism, such as the long term impact, are still not clear and need to be further investigated.

The above studies focus on the energy market, which is only part of the auctions conducted by ISOs. Besides energy, ancillary services (e.g., regulation, spinning reserve and non-spinning reserve) are also important products. The ancillary service bids can be either optimized after the energy auction, or co-optimized simultaneously with energy bids. Most ISOs co-optimize energy and ancillary service bids, since co-optimization leads to a more efficient allocation of limited capacities than optimizing each product individually.

Study on co-optimization is needed for a thorough comparison of the two mechanisms. In view that prices are decision variables in the objective function of PCM, they need to be clearly defined for each product. The total payment cost function is discontinuous, since payments can suddenly jump to a higher level when a more expensive unit begins to set the price. As a result, the payment cost function is non-differentiable at those break points, and the prices cannot be defined by taking partial derivatives of the Lagrangian of Economic dispatch problem as what is commonly used in the BCM price definition. The price definition in payment cost minimization needs to be defined in a different way.

Solving PCM problem is also more challenging than solving BCM problems. The BCM problems can be solved efficiently in ISOs by using standard MIP solvers, which requires problem linearity. The payment cost function, however, is nonlinear with a cross-product of prices and generation/spinning reserve levels, and cannot be directly solved by the branch-and-cut solvers. The problem is complicated in co-optimization by the coupling of energy and ancillary services through unit capacity constraints. Few results have been reported for payment cost co-optimization yet, which makes the comparison of the two mechanisms extremely difficult.

This paper formulates and solves the payment cost co-

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optimization in a day-ahead market. Literature on solving single-product optimization and co-optimization problems under two mechanisms is reviewed in Section II. Our problem formulation is presented in Section III. As mentioned earlier, the partial derivative or Lagrangian cannot be used in PCM price definition. To define the price in PCM, we introduce the marginal candidate set concept, which defines a set of potential price setting units. The marginal unit is the most expensive unit in the set and will set the price. This is consistent with the marginal pricing theory. For problems with transmission capacity constraints, determining the marginal candidate set for each bus is difficult due to the congestion. However, the marginal candidate units located at each bus still form a subset of the marginal candidate set of that bus, and the LMP at that bus can be no lower than any of the bid prices in the subset. The LMP difference between an arbitrary bus and the reference bus is then derived by using KKT conditions of the Lagrangian.

The PCM problems are nonlinear, and cannot be solved directly by the branch-and-cut based MIP solvers. This difficulty still exists when decomposed into subproblems. To deal with nonlinearity, an iterative method based on our previous work on Lagrangian Relaxation and surrogate subgradient optimization is developed in this paper. The overall PCM problem is decomposed into individual subproblems and solved iteratively such that at each step, the subproblem model has a linear formulation. A two-level structure similar to Lagrangian Relaxation is used: Updating multipliers at the high level and solving subproblems at the low level. To deal with the nonlinearity created by the cross-product of price and individual generation/spinning reserve level, the prices are fixed when subproblems are solved. Based on the subproblem solutions, the prices are then systematically updated by using price definition constraints. The surrogate condition is checked every iteration to guarantee a proper direction for updating multipliers. An estimate of L^* is used when calculating the stepsize. Further investigation is needed to avoid the divergence of algorithm caused by overestimate. Details of the new method are presented in Section IV.

Numerical results presented in Section V demonstrate the correctness of the new price definition on small examples, and the effectiveness and efficiency of our new iterative method based on 24-bus transmission co-optimization problems.

II. Literature Review

Prices appear explicitly in the objective function as decision variables in PCM and they need to be appropriately defined and to be operationalized throughout the optimization. The prices are defined as the highest bid price among the selected bids in [1][3]. This "highest price" definition is correct except for the case that expensive units must generate at p_{min} to satisfy the system demand. The "partial derivative" definition uses the partial derivative of Lagrangian to define prices. It does not work in PCM, since no partial derivative exists at those break points when a new unit starts to set prices. To avoid this, the total payments are substituted by total bid costs in the Lagrangian of economic dispatch in [6] when deriving the partial derivatives. This substitution, however, leads to inconsistency between the unit commitment and economic dispatch. A consistent price definition for payment cost minimization is desired.

Both PCM and BCM belong to the mixed integer programming (MIP) problems. The Lagrangian Relaxation technique and branch-and-cut method are two typical methods used for solving such problems. In the Lagrange Relaxation framework, the multipliers can provide clear economic interpretation as the shadow prices associated with the constraints, and the relaxed problem can be decomposed to individual sub-problems if the problem is separable. According to the separability and the roles of prices, the use of Lagrange Relaxation techniques can be divided into three categories. The first category consists of separable problems which can be decomposed into individual subproblems after relaxation. One typical example for such problems is the machine scheduling problem in which the multiplier relaxing capacity constraints provides quick solution to the "what if" questions [4]. The bid cost minimization problems are also separable problems. The second category consists of non-separable problems in which prices are not decision variables, e.g., problems with non-linear coupling constraints. The third category is non-separable problems in which prices are decision variables. Payment cost minimization is a typical example for such problems. An augmented Lagrangian and surrogate optimization framework for solving payment cost minimization problems was presented in [3]. It is proved that if surrogate condition is satisfied, a proper direction for updating multipliers can be obtained [14]. The conditions on stepsize and convergence are presented in [5]. Based on [3], transmission capacity constraints are incorporated [6].

The branch-and-cut method is an alternative for solving MIP problems. In branch-and-cut, integrality constraints are first relaxed. Valid cuts maintaining all the feasible solutions are generated to tighten the bound of the continuous relaxation to obtain the convex hull of the original feasible solutions. The linear programming (LP) simplex method then efficiently optimizes the relaxed LP problem over the convex hull and an optimal solution can be obtained, which is also the optimal solution to the original problem in view of problem linearity. Since obtaining the convex hull itself is NP hard, branching operations may be needed to decompose the problem as in the branch-and-bound method. According to problem linearity and constraint structures, the mixed MIP can also be divided into the following three categories. The first category consists of linear problems with "tight" formulations. Either the continuous relaxation can provide a good approximation of the convex hull, or the structure of constraint sets can be used to generate strong facet-defining cuts to approximate the convex hull. The second category consists of linear problems with "loose" formulations. The feasible regions of such problems are enlarged drastically after relaxing integrality constraints and their continuous relaxations provide poor lower bounds. Issues about tight linear formulation for mixed integer programming are discussed in [7]. The third category consists of problems with non-linear formulation, with product terms or logical expressions in the constraints or objective functions. PCM problems are typical examples in this category. They are difficult to be solved effectively unless properly converted to linear forms.

The studies on multi-product bid cost optimization can be divided into two categories: sequential optimization and simultaneous optimization. The sequential auction for ancillary services that is used in the California ISO (CAISO) is described in [8] and [9]. For simultaneous optimization, the co-optimization model for simultaneous auctions that is used in the New York ISO (NYISO) and New England ISO (ISO-NE) are described in [10] and [11]. A detailed AC OPF-based formulation for procuring, pricing, and settling energy and ancillary service is presented in [12], with the economic dispatch problem solved as a LP. The payment cost co-optimization, however, is seldom addressed.

III. Problem formulation

In this section, the payment cost co-optimization problem is formulated for a day-ahead energy market with given energy demand and spinning reserve requirement. The formulation of the co-optimization problem presented in subsection III-A. The price definitions for energy and spinning reserve products are presented in subsection III-B. Details of linear conversion are presented in subsection III-C.

A. Problem formulation

Consider a transmission network with I buses connected by L transmission lines. For each bus i, there are K_i supply units indexed by k. Each unit submits both supply energy bid and spinning reserve bid. The generation level and spinning reserve level of the ik^{th} bid is denoted by $p_{ik}^E(t)$ and $p_{ik}^S(t)$, respectively. $p_{ik}^E(t)$ is limited by the minimum generation level p_{ikmin}, the maximum generation level p_{ikmax}. $p_{ik}^S(t)$ can be allocated starting from 0 and cannot exceed the maximum spinning reserve level p_{ikmax}^S. The startup cost is denoted by S_{ik} and is incurred if and only if the supply bid is turned "on" from "off" at hour t. The energy price for bus i at hour t is denoted by $LMP_i^E(t)$. The spinning reserve price at hour t is denoted by $MCP^S(t)$. The co-optimization problem is to minimize the total payment costs subject to system demand constraints, spinning reserve requirements, transmission capacity constraints, individual unit constraints, and the price definition constraints.

System constraints:

System demand constraints: The total power generation should be equal to the system demand at hour t:

$$\sum_{i=1}^{I}\sum_{k=1}^{Ki} p_{ik}^E(t) = \sum_{i=1}^{I} P_i^{DE}(t), \ \forall t. \tag{1}$$

System spinning reserve requirements: The total spinning reserve level should be equal to the system spinning reserve requirements at hour t:

$$\sum_{i=1}^{I}\sum_{k=1}^{Ki} p_{ik}^S(t) = P^{DS}(t), \ \ \forall t. \tag{2}$$

DC power flow equations: The flow $f_l(t)$ in line l at hour t can be expressed as the linear combination of net nodal injection of energy [13] at hour t:

$$f_l(t) = \sum_{i=1}^{I}\left[a^i{}_l \cdot \left(\sum_{k=1}^{Ki} p_{ik}^E - P_i^{DE}(t)\right)\right], \forall l, \forall t. \tag{3}$$

The shift factor $a^i{}_l$ denotes the sensitivity of the transmitted power in line l with respect to the net injection at bus i. It is determined by the network structure and physical parameters of transmission lines.

Transmission capacity constraints: The flow in line l cannot exceed the transmission capacity in any hour, i.e.,

$$-f_{l\max} \le f_l(t) \le f_{l\max}, \forall l, \forall t. \tag{4}$$

The transmission capacity limits for both directions are set to be the same for simplicity.

Equation (3) and (4) are combined together in our problem formulation and equation (5) is used.

$$-f_{l\max} \le \sum_{i=1}^{I}\left[a^i{}_l \cdot \left(\sum_{k=1}^{Ki} p_{ik} - P_i^D(t)\right)\right] \le f_{l\max}, \forall l, \forall t. \tag{5}$$

Individual unit constraints:

Unit capacity constraints: The sum of generation and spinning reserve level of a unit must be within its minimum and maximum limits if it is committed. The generation level and spinning reserve level are both 0 if the unit is not selected. The relationship can be expressed as:

$$x_{ik}(t) p_{ik\min} \le p_{ik}^E(t) + p_{ik}^S(t) \le x_{ik}(t) p_{ik\max}, \forall i, \forall k, \forall t. \tag{6}$$

Energy capacity constraints: The committed generation level must be within the minimum and maximum limits of the unit, if the capacity is allocated to the energy market, i.e., the allocation status variable $x_{ik}^E = 1$. If $x_{ik}^E = 0$, then the capacity of the unit is not allocated to the energy market.

$$x_{ik}^E(t) p_{ik\min} \le p_{ik}^E(t) \le x_{ik}^E(t) p_{ik\max}, \forall i, \forall k, \forall t. \tag{7}$$

Spinning reserve capacity constraints: The selected spinning reserve level must be within the 0 and maximum limits if the capacity is allocated to the spinning reserve market, i.e., $x_{ik}^S = 1$. If $x_{ik}^S = 0$, then the capacity of the unit is not allocated to the spinning reserve market. To ensure that $x_{ik}^S = 0$ for unselected reserve bids, a small positive number δ is used as the minimal limit. The relationship can be expressed as:

$$x_{ik}^S(t) \cdot \delta \le p_{ik}^S(t) \le x_{ik}^S(t) p_{ik\max}^S, \forall i, \forall k, \forall t. \tag{8}$$

Startup cost constraint: The binary decision variable $u_{ik}(t)$ denotes the "turn on" decision for unit ik at hour t. The variable $u_{ik}(t)$ takes 1 if and only if the supply unit is turned "on" from "off" at hour t, i.e., only if the binary status variable $x_{ik}(t\text{-}1)$ equals 0 and $x_{ik}(t)$ equals 1. This relationship can be expressed linearly in equation (9):

$$u_{ik}(t) \ge x_{ik}(t) - x_{ik}(t-1), \forall i, \forall k, \forall t \ . \tag{9}$$

TABLE I
Relationship between status variable x and decision variable u_{ik}

$x_{ik}(t)$	$x_{ik}(t\text{-}1)$	$u_{ik}(t)$
0	0	$u_{ik}(t) \ge 0$ (redudent)
0	1	$u_{ik}(t) \ge -1$ (redudent)
1	0	$u_{ik}(t) \ge 1$ (fixed at 1)
1	1	$u_{ik}(t) \ge 0$ (redudent)

Table 1 describes the above relationship. It can be seen that when $x_{ik}(t\text{-}1)$ equals 0 and $x_{ik}(t)$ equals 1, $u_{ik}(t)$ is fixed at 1, otherwise equation (9) will be redundant.

B. Price definition

LMP_i^E definition:

As mentioned earlier, if some expensive unit has to generate at its p_{min} to satisfy the system demand constraints, such expensive unit is not a price-setting unit although online, and should be excluded from the set of potential price setting units, or "marginal candidate set". Other online units are candidates for setting prices and they are in the marginal candidate set. The price-setting marginal unit is the one with the highest bid price in the set and will set the prices. To characterize the marginal candidate set, a binary variable y_{ik}^E is introduced to denote whether a unit is in the set or not. The logical relationship between the on/offline status variable x_{ik} and the marginal candidate status variable y_{ik}^E forms the condition for marginal candidate set, and can be analyzed as follows:

If $x_{ik} = 0$, then $y_{ik}^E = 0$, i.e., offline units will not set prices;

If $x_{ik} = 1$, then $y_{ik}^E = 0$ only if the unit must generate at its p_{min} to satisfy the system demand. Otherwise, $y_{ik}^E = 1$ for online units.

A unit must generate at p_{min} if the available online capacity provided by other units cannot cover the amount of power provided by the p_{min} unit, i.e., $\sum_{jh\neq ik}(p_{jhmax}\cdot x_{jh} - p_{jh}^E) < p_{ikmin}$.

When considering transmission capacity constraints, the marginal candidate set for each bus cannot be easily identified due to the transmission congestion. However, the units located at a bus i with y_{ik}^E=1 still form a subset of the marginal candidate set of bus i, thus the following equation holds:

$$LMP_i^E \geq c_{ik}^E \cdot y_{ik}^E \tag{10}$$

The above constraint is active when a local unit ik is setting prices for bus i. If LMP_i^E is set by a remote unit located at other buses, it means that any cheap capacity located at i has been exhausted and the LMP_i^E set by a remote units must be higher than any bid prices in the local marginal candidate subset formed by units with y_{ik}^E=1, i.e., $LMP_i^E > c_{ik}^E \cdot y_{ik}^E$. To determine the LMPs, the KKT conditions are derived for the relaxed problem.

Constraints (1), (2) and (5) are system-wide coupling constraints. By using multipliers $\{\lambda^E\}$ to relax the system demand constraints, using multipliers $\{\lambda^S\}$ to relax the spinning reserve requirement constraints, using multipliers $\{\gamma_{max}, \gamma_{min}\}$ to relax the transmission capacity limit for line l along the positive and negative direction, the relaxed problem can be obtained in equation (11).

$$\begin{aligned}
\min: L(x,\lambda) = &\sum_{i=1}^{I} LMP_i^E \cdot \sum_{k=1}^{Ki} p_{ik}^E + MCP^S \cdot \sum_{i=1}^{I}\sum_{k=1}^{Ki} p_{ik}^S \\
&+ \lambda^E \cdot \left[\sum_{i=1}^{I} P_i^{DE} - \sum_{i=1}^{I}\sum_{k=1}^{Ki} p_{ik}^E\right] \\
&+ \lambda^S \cdot \left[P^{DS} - \sum_{i=1}^{I}\sum_{k=1}^{Ki} p_{ik}^S\right] \\
&+ \sum_{l=1}^{L}\gamma_{l\max} \cdot \left[\sum_{i=1}^{I} a^i{}_l \cdot \left(\sum_{k=1}^{Ki} p_{ik} - P_i^D\right) - f_{l\max}\right] \\
&+ \sum_{l=1}^{L}\gamma_{l\min} \cdot \left[-\sum_{i=1}^{I} a^i{}_l \cdot \left(\sum_{k=1}^{Ki} p_{ik} - P_i^D\right) - f_{l\max}\right]
\end{aligned} \tag{11}$$

subject to individual unit constraints (6)-(9).

KKT conditions of the relaxed problem: p_{ik}^E and p_{ik}^S are the decision variables in the relaxed problem. For energy product, if unit ik is setting the energy price, its optimal generation level p_{ik}^{E*} must be inside the region formed by equations (6)-(9), i.e., constraints (6)-(9) are not active constraints. The KKT conditions can thus be derived by taking partial derivative of (11) with respect to p_{ik}^E:

$$\frac{\partial L}{\partial p_{ik}^E} = LMP_i^E - \lambda^E + \sum_{l=1}^{L}(\gamma_{l\max} - \gamma_{l\min}) \cdot a^i{}_l = 0 \tag{12}$$

$$\gamma_{l\max} \geq 0 \tag{13}$$

$$\gamma_{l\min} \geq 0 \tag{14}$$

For the reference bus, a_l^i =0, thus we have

$$LMP_{ref}^E = \lambda^E \tag{15}$$

The relationship between an arbitrary LMP_i^E and LMP_{ref}^E can thus be obtained.

$$LMP_i^E = LMP_{ref}^E + \sum_{l=1}^{L}(\gamma_{l\min} - \gamma_{l\max}) \cdot a^i{}_l = 0 \tag{16}$$

Equation (10), (13),(14) and (16) are used in our model to define the energy price.

MCP^S definition:

As mentioned in Section II, the spinning reserve product can be allocated from 0, thus there is no "p_{min} issues" for spinning reserve and the highest price definition can be used. By using binary variable x_{ik}^S to denote the allocation status, the spinning reserve price can be defined as follows

$$MCP^S \geq c_{ik}^S \cdot x_{ik}^S \tag{17}$$

C. Linear conversion

The logical constraints on marginal candidate status variable y_{ik}^E can be summarized in the following two cases:

Case 1: Online units with $x_{ik} = 1$

$y_{ik}^E = 0$, if $p_{ik}^E = p_{ikmin}$ and $\sum_{jh\neq ik}(p_{jhmax}\cdot x_{jh} - p_{jh}^E) < p_{ikmin}$

$y_{ik}^E = 1$, if $p_{ik}^E > p_{ikmin}$, or $p_{ik}^E = p_{ikmin}$ but $\sum_{jh\neq ik}(p_{jhmax}\cdot x_{jh} - p_{jh}^E) \geq p_{ikmin}$

In the first sub-case, unit ik must generate at p_{min} since the remaining online capacity by other units cannot satisfy the system demand. Such p_{min} unit is not considered as a marginal unit candidate, i.e., online but will not set prices. Other online units are included in the second sub-case and they will be considered as a candidate for setting prices.

Case 2: Offline units with $x_{ik} = 0$

$y_{ik}^E = 0$, i.e., offline units will not set the price.

In view of minimization, binary variable y_{ik}^E will be minimized to 0 unless it is fixed at 1. It can be seen that only in the second sub-case in case 1 is y_{ik}^E fixed at 1. The following two linear constraints fix y_{ik}^E at 1 only when the conditions in the second sub-case are satisfied, and are redundant in other cases, so that y_{ik}^E will be minimized to 0.

$$y_{ik}^E \geq \frac{p_{ik}^E - p_{ik\min}}{p_{ik\max} - p_{ik\min}} \tag{18}$$

$$y_{ik}^{E} \geq \frac{\sum_{jh \neq ik}\left(p_{jh\max} \cdot x_{jh} - p_{jh}^{E}\right) - p_{ik\min} + \varepsilon}{M} + (x_{ik} - 1) \quad (19)$$

where ε is a small positive number and M is a big positive number.

To see this, the second sub-case in case 1 is checked first. If unit ik is online and $p_{ik}^{E} > p_{ikmin}$, then y_{ik}^{E} is fixed at 1 by equation (18). If $p_{ik}^{E} = p_{ikmin}$ but $\sum_{jh\neq ik}(p_{jhmax} \cdot x_{jh} - p_{jh}^{E}) \geq p_{ikmin}$, then equation (18) is redundant. $\sum_{jh\neq ik}(p_{jhmax} \cdot x_{jh} - p_{jh}^{E}) \geq p_{ikmin}$ can be expressed as $\sum_{jh\neq ik}(p_{jhmax} \cdot x_{jh} - p_{jh}^{E}) + \varepsilon > p_{ikmin}$, so the right hand side of equation (19) is positive, fixing y_{ik}^{E} to be 1. In all the other cases, both (18) and (19) are redundant and y_{ik}^{E} will be minimized to 0.

Objective function:

With startup costs fully compensated, the total payment cost is the sum of MW payments and startup compensations across the system over the 24 hour period, i.e.,

$$J \equiv \sum_{t=1}^{T}\left\{\sum_{i=1}^{I}[LMP_{i}^{E}(t) \cdot \sum_{k=1}^{Ki} P_{ik}^{E}(t)] + MCP^{S}(t) \cdot \sum_{i=1}^{I}\sum_{k=1}^{Ki} P_{ik}^{E}(t)\right\} + \sum_{t=1}^{T}\sum_{i=1}^{I} u_{ik}(t) S_{ik}\,. \quad (20)$$

IV. Solution Methodology

As mentioned earlier, the payment cost minimization problem is nonlinear, with a cross-product of prices and generation/spinning reserve levels. Such problem cannot be directly solved by the branch-and-cut based solvers. We recently presented a method to decompose the overall problem into subproblems based on Lagrangian Relaxation and surrogate optimization technique [5], but the subproblems considered in that paper are all linear. In our problem, the nonlinear cross-product still exists in the subproblem formulations. To deal with nonlinear subproblems, a method is developed based on our previous work to iteratively solve the subproblems such that at each step the model is still linear. The decomposition and subproblem formulation are presented in subsection IV-A. The method for solving nonlinear subproblems is presented in IV-B. The multipliers are updated at the high level and details are presented in IV-C.

A. Decomposition and subproblem formulation

As mentioned earlier in Section III, the relaxed problem is (11), subject to constraints on individual units. The relaxed problem can be decomposed into individual unit subproblem. For each unit ik, the subproblem is

$$\min L_{ik}, with L_{ik} = \left[LMP_{i}^{E} - \lambda^{E} + \sum_{l=1}^{L}(\gamma_{l\max} - \gamma_{l\min}) \cdot a_{l}^{i}\right] p_{ik}^{E} + \left(MCP^{S} - \lambda^{S}\right) p_{ik}^{S}$$

s.t.

Unitcapacity: $x_{ik}(t) p_{ik\min} \leq p_{ik}^{E}(t) + p_{ik}^{S}(t) \leq x_{ik}(t) p_{ik\max}, \forall t.$

Product capacity: $x_{ik}^{E}(t) p_{ik\min} \leq p_{ik}^{E}(t) \leq x_{ik}^{E}(t) p_{ik\max}, \forall t.$

$x_{ik}^{S}(t) \cdot \delta \leq p_{ik}^{S}(t) \leq x_{ik}^{S}(t) p_{ik\max}^{S}, \forall t.$

Startup cost: $u_{ik}(t) \geq x_{ik}(t) - x_{ik}(t-1), \forall t$

The price definition constraints are used for updating prices iteratively after the subproblems are solved. They are not considered as constraints for subproblems.

The overall problem can be solved by using a 2-level structure similar to the standard Lagrangian Relaxation: At the high level, multipliers are updated based on the subproblem solutions. At the low level, the subproblems are solved with given multipliers. To get rid of nonlinearity of the subproblems, we develop a method to solve it iteratively. Details of the two levels will be elaborated in the following.

B. Updating multipliers at the high level

At the high level, the multipliers are updated based on the newly obtained subproblem solutions. To update multipliers, several methods have been presented. We use the standard subgradient method. The convergence of the method has been proved in [14] [5].

In our method, the multiplier λ^{E} relaxing system demand constraints can be updated according to the following equation.

$$\lambda^{E(k+1)} = \lambda^{Ek} + c^{k} g(\lambda^{Ek}) \quad (21)$$

where k is the index for iterations, $c^{k} = \alpha \frac{L^{*} - L^{k}}{\left\|g(\lambda^{k})\right\|^{2}}$ is the stepsize for updating multipliers at iteration k, and $g(\bullet)$ is the subgradient of the dual function with respect to certain dual variables. According to the relaxed problem, we have

$$g(\lambda^{E}) = \sum_{i=1}^{I} P_{i}^{DE} - \sum_{i=1}^{I}\sum_{k=1}^{Ki} p_{ik}^{E} \quad (22)$$

Similarly, the equations for updating other multipliers are given as follows:

$$\lambda^{S(k+1)} = \lambda^{Sk} + c^{k} g(\lambda^{Sk}) \quad (23)$$

where $g(\lambda^{S}) = P^{DS} - \sum_{i=1}^{I}\sum_{k=1}^{Ki} p_{ik}^{S}$

$$\gamma_{l\max}^{k+1} = \gamma_{l\max}^{k} + c^{k} g(\gamma_{l\max}^{k}) \quad (24)$$

where $g(\gamma_{l\max}) = \sum_{i=1}^{I} a^{i}{}_{l} \cdot \left(\sum_{k=1}^{Ki} p_{ik} - P_{i}^{D}\right) - f_{l\max}$

$$\gamma_{l\min}^{k+1} = \gamma_{l\min}^{k} + c^{k} g(\gamma_{l\min}^{k}) \quad (25)$$

where $g(\gamma_{l\min}) = -\sum_{i=1}^{I} a^{i}{}_{l} \cdot \left(\sum_{k=1}^{Ki} p_{ik} - P_{i}^{D}\right) - f_{l\max}$

Generally, the optimal value L^{*} is unknown and an estimated value of L^{*} is currently used in our method.

C. Solving subproblems iteratively at the low level

As mentioned earlier, the subproblem objective function is still nonlinear. Although the multipliers are fixed when solving subproblems, the cross-product of prices and generation/spinning reserve levels still exists. To get rid of the nonlinear term, we develop a method to solve subproblems iteratively by using the standard branch-and-cut based solvers.

At iteration k, the prices are first fixed at their values at the k-1 iteration such that the subproblem formulation is linear.

The modified subproblem is:

$$\min_{p,x} L_i = \left[LMP_i^{E(k-1)} - \lambda^{Ek} + \sum_{l=1}^{L} (\gamma_{l\max}{}^k - \gamma_{l\min}{}^k) \cdot a_l^i \right] p_{ik}^{Ek} + \left(MCP^{S(k-1)} - \lambda^{Sk} \right) p_{ik}^{Sk} \quad (26)$$

The subproblem is then solved for generation/spinning reserve levels p_{ik}^E/p_{ik}^S, allocation status x_{ik}^E/x_{ik}^S, and commitment status x_{ik}.

After solving each subproblem, the prices are pre-updated based on the price definition constraints. The energy price LMP_i^E are updated according to (10) and (16)

Note that equation (16) is derived at the optimal point, but may not hold at any iteration k along the process. In practice, we calculate the flows based on subproblem solutions using DC flow equations, and then determine the sign of γ_{lmax} and γ_{lmin}. The *LMPs* are then selected to be the minimal values that satisfy all the constraints on LMP, γ_{lmax} and γ_{lmin}.

The spinning reserve price is defined to be the highest price among the selected spinning reserve bids. Given subproblem solutions, the spinning reserve price can be determined by (17), or can be written as $MCP^S = \max \left\{ c_{ik}^S \cdot x_{ik}^S \right\}, \forall i, \forall k$. To guarantee the convergence of the algorithm, both the updating direction and the stepsize need to satisfy certain conditions. It has been proved that if the surrogate condition

$$L(\lambda^{k+1}, x^{k+1}) < L(\lambda^{k+1}, x^k) \quad (27)$$

is satisfied, the direction for updating the multiplier forms an acute angle with the direction towards the optimal value of the multiplier, and multipliers move closer to their optimal values iteration by iteration. The conditions on stepsize are presented in [14] and [5]. Since L^* is generally unknown, an estimated value is used in our algorithm. If the L^* is overestimated too much, then the conditions on stepsize may not be satisfied, and the algorithm may diverge. In our algorithm, the surrogate condition is checked after each subproblem is solved to guarantee a proper direction for updating multipliers, and L^* is estimated by using heuristics presented in [6]. If the surrogate condition is not satisfied, the subproblem solutions remain their values at previous iteration.

After all the subproblems are solved, the LMP_i^E and MCP^S will be updated according to the price definition constraints. A new set of subproblem solution consisting of p_{ik}^E, p_{ik}^S, x_{ik}^E, x_{ik}^S, x_{ik}, LMP_i^E and MCP^S is then obtained and will be used for updating multipliers in the next iteration.

V. NUMERICAL RESULTS

Our iterative method has been run on an Intel Xeon dual 1.6-GHz server with 8G memory. Three different size co-optimization examples are tested. Example 1 uses a 1-hour example to examine the marginal candidate set conditions, and compares the results obtained in CPLEX MIP with our analysis to verify the correctness of the price definition. Example 2 uses a 2-bus example modified based on the 2nd example in [1] to examine the solution quality of our new method in terms of multipliers and subproblem solutions. Example 3 then tests the algorithm on a 24-h co-optimization problem in a 24-bus network to demonstrate the efficiency and effectiveness of our method.

Example 1: Consider a 3-bid 1-hour co-optimization problem. Each supplier submits energy bid and spinning reserve bid. The parameters of the bids are given in table II. The energy demand is 100MW. The spinning reserve requirement is set to be 5% of the energy demand and is 5MW. The startup costs are set to be 0.

TABLE II
PARAMETERS OF THE THREE UNITS

	Energy Price ($/MW)	SR Price ($/MW)	p_{min} (MW)	p_{max} (MW)	Startup cost ($)
Unit 1	10	5	0	30	0
Unit 2	70	25	40	60	0
Unit 3	80	30	40	50	0

The testing results are listed in table III. Since the energy demand is much higher than the spinning reserve requirements, cheap capacity will be allocated to the energy market first. Unit 1 and unit 2 are cheaper than unit 3 and will be first selected. In view of the high p_{2min} and p_{3min}, both unit 1 and unit 2 need to reduce their generation levels to meet the energy demand. As a result, unit 1 will generate at 20MW and will set MCP^E to be 10$/MW. Both unit 2 and unit 3 will generate at their minimal generation limits to satisfy the energy demand. 5MW of the remaining cheap capacity from unit 1 will be allocated to the spinning reserve market to set MCP^S to be 5$/MW.

The conditions for marginal candidate set can be examined. Unit 2 should be excluded from the marginal candidate set since the available online capacity from unit 1 and unit 3 (10 + 10 = 20MW) is smaller than p_{2min} (40MW). Similarly, unit 3 should also be excluded from the marginal candidate set. In our price definition, $y_2^E = y_3^E = 0$, i.e., they are online but will not set the prices. As a result, unit 1 will be the only one in the marginal candidate set and set MCP^E to be 10$/MW. Unit 1 is the only one allocated to spinning reserve market and sets MCP^S to be 5$/MW. The total payment is $1025.

TABLE III
TESTING RESULTS FOR THE THREE-UNIT 2-PMIN PROBLEM

	p_{ik}^E	p_{ik}^S
Unit 1	20	5
Unit 2	40	0
Unit 3	40	0
MCP^E($)	10	
MCP^S($)	5	
Payment ($)	1025	

Example 2: Consider a 2-bid 2-bus co-optimization problem based on the second example in [1]. The transmission network is shown in Fig. 1. The two units are located at bus 1 and bus 2, respectively. The parameters of the two units are given in table IV. The 1-h energy demands are 60MW at bus 1, and 40MW at bus 2. The spinning reserve requirement for the whole system is set to be 5% of the total energy demands and is 5MW. Bus 2 is the reference bus.

Without congestion, the example reduces to the 2nd example in [1] and $p_{11}^E = 100$, $p_{11}^S = 0$, $p_{21}^E = 0$, $p_{21}^S = 5$. The flow in the non-congestion case can thus be calculated to be (100-60) = 40MW. The transmission capacity of the line is thus set to be 30MW to create the congestion. The congestion example is solved by our algorithm, and the optimal solution is $p_{11}^E = 90$, $p_{11}^S = 5$, $p_{21}^E = 10$, $p_{21}^S = 0$, $LMP_1^E = 20$, $LMP_2^E = 25$, $MCP^S = 2$.

TABLE V
PARAMETERS OF UNITS IN EXAMPLE 2

	Energy bid price ($/MW)	Spinning reserve bid price ($/MW)	p_{min} (MW)	p_{max} (MW)	p_{max}^S (MW)	Startup cost ($)
Unit 11	20	2	0	100	6	0
Unit 21	25	8	0	10	6	0

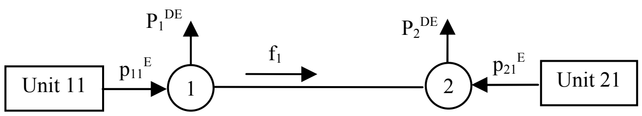
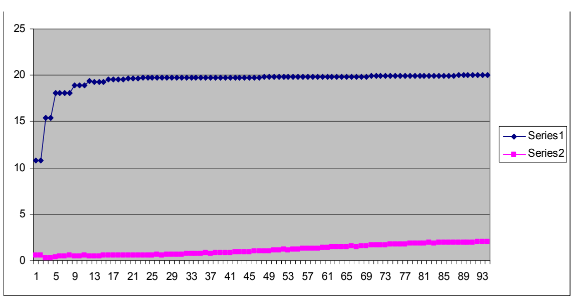


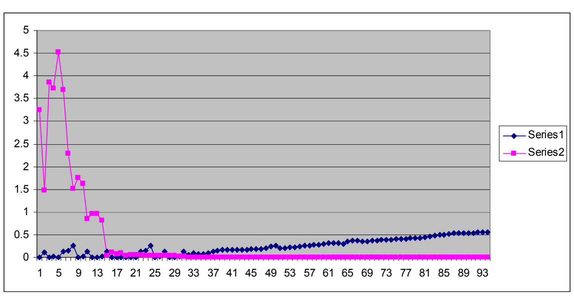
Fig. 1. The network structure of example 2

The convergence of multipliers is shown in Fig. 2. It can be seen that λ^E converges to 20. This can be explained as follows. Suppose 1 additional MW is needed from this 2-bus system, unit 11 will be responsible for that, since it is cheaper and unit 21 is at its p_{max}. As a result, the λ^E converges to 20, which is the bid price of unit 11. λ^S converges to 2, which is the MCP^S set by unit 11. The γ_{lmin} reduces to 0 gradually and γ_{lmax} converges to certain positive value, indicating the congestion along the positive direction.

The subproblem solution oscillates before convergence. The λ^E, λ^S and L^k are selected to plot a 3-D figure in Fig. 3, showing the convergence track from three different views. It can be seen that multipliers jump on the facets in the dual space and converge to $L^* = 2060$.



(a) The convergence of λ^E and λ^S



(b) The convergence of γ_{lmax} and γ_{lmin}

Fig. 2. The convergence of multipliers: (a) λ^E and λ^S. (b) γ_{lmax} and γ_{lmin}.

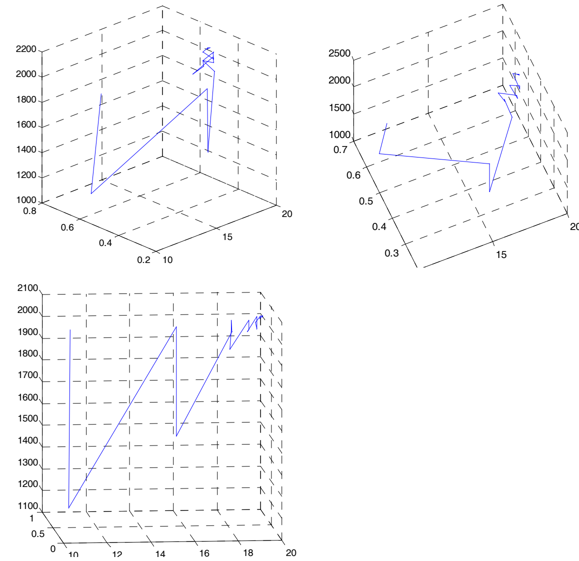


Fig. 3. The convergence track

Example 3: Consider a 24-hour co-optimization problem in a 24-bus system. The example is build based on the IEEE Reliability Testing System – 1996 (IEEE RTS-96). The unit parameters are set to be the same as the example presented in IEEE RTS-96. As presented in [15], the whole system is divided into two zones. There is 46.74% of system load in zone I, but the total generation capacity located in zone I is only 20.09% of the whole system. The power is thus delivered from zone II to zone I. The line limit of the five lines connecting zone I and zone II are reduced to create congestion, while the capacities for other lines are set to be big enough. The supply bid parameters are set to be the same as [15]. The system demand is randomly generated with Gaussian distribution based on load-data. The spinning reserve requirement is set to be equal to 5% of the total system demand at any hour.

The problem is solved by our algorithm. The default setting is used for all the CPLEX MIP parameters. The algorithm converges after 736s, obtaining a set of multipliers. Given these multipliers, a feasible solution is obtained by heuristics, with total payment costs $\$1.612 \times 10^6$. Further investigation is needed to tune up the performance and to guarantee the convergence.

VI. Conclusion

The co-optimization of energy and spinning reserve under payment cost minimization is discussed in this paper. The price for each product is defined to be the marginal bid price among the units providing that product. Co-optimization problems under the PCM set up is nonlinear, and cannot be solved directly by the branch-and-cut based solvers, which requires problem linearity. Based on our previous work on Lagrangian Relaxation and surrogate subgradient optimization, we develop a new method to deal with nonlinearity. The prices are first fixed at their values at the previous iteration to obtain a linear formulation and are then updated based on subproblem solutions by using price definition constraints if surrogate condition is satisfied. This method provided one way to solve nonlinear problems which cannot be converted to linear by using the branch-and-cut based MIP solvers. It can be used not only for payment cost co-optimization, but also for solving other mixed integer programming problems with similar structures.

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